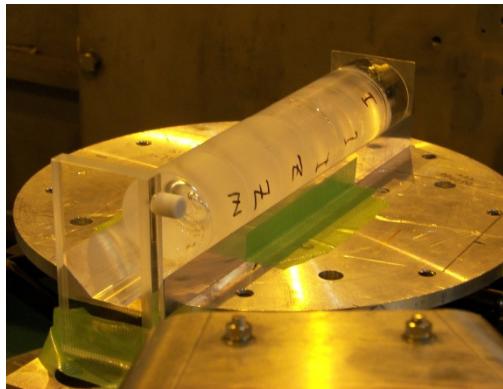


# **Resolution Function of SANS Diffractometer with Refractive Lenses**

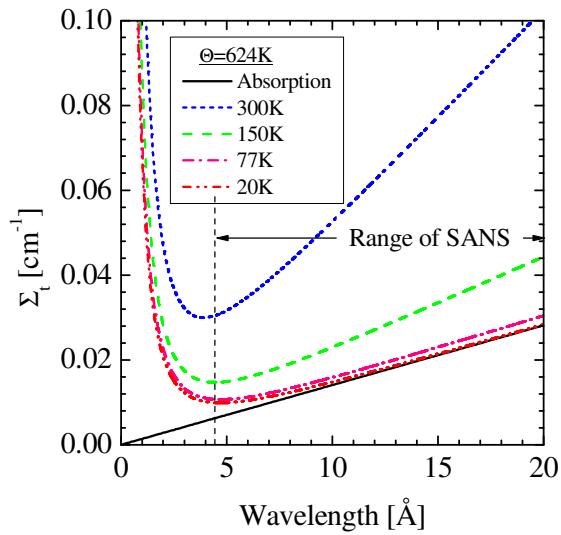
H. Frielinghaus  
F. Lipfert



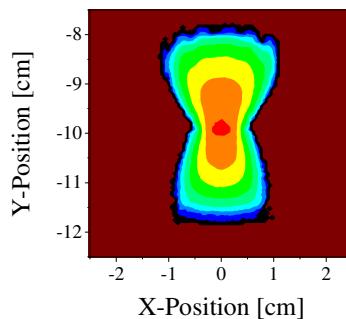
# Components: Neutron Lenses



Purchased from Zeiss  
and Ingeneric (Aachen)  
full sets KWS1/2 (2x26)



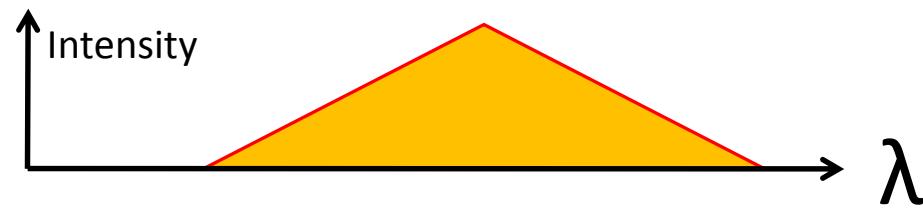
Effects of phonon scattering calculated  
Lenses will be cooled to ~70K  
Cooled lens holder in construction.



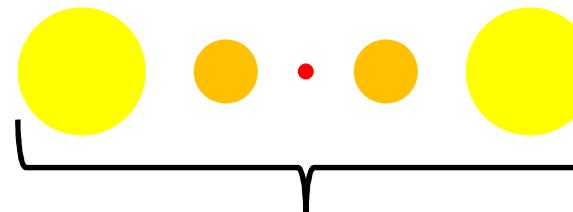
Simulation routines for  
McStas and Vitess existing.

# Resolution of Neutron Lenses

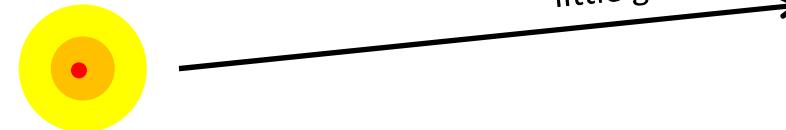
Wavelength Distribution from Selector



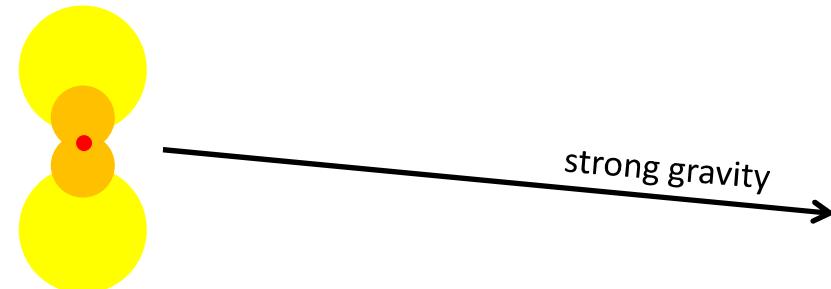
Focus:



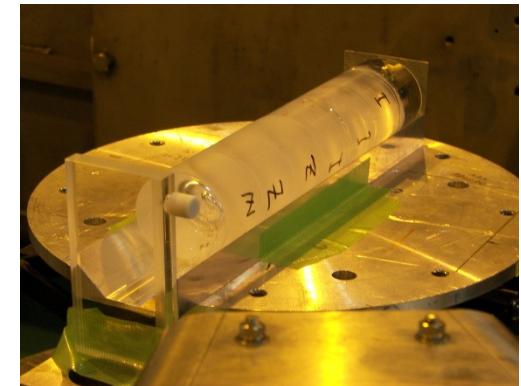
No Gravity:



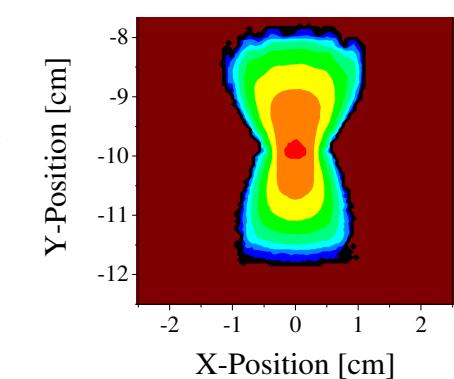
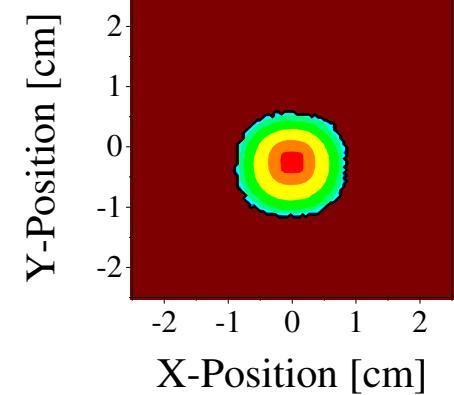
With Gravity:  
(peanut)



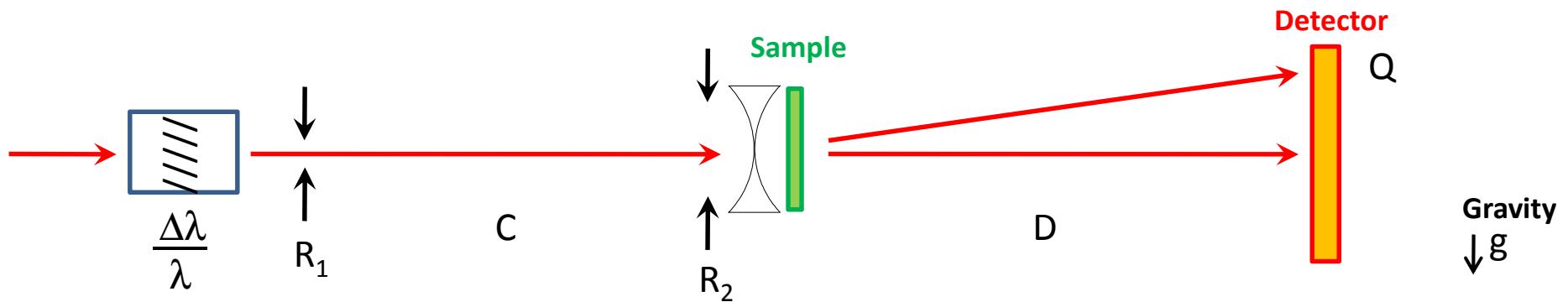
Strength of this effect  $\sim \lambda^2 D^2$  i.e. strongest for low Q.



Simulations:



# 1<sup>st</sup> step of resolution correction: isotropic, rad. averaged



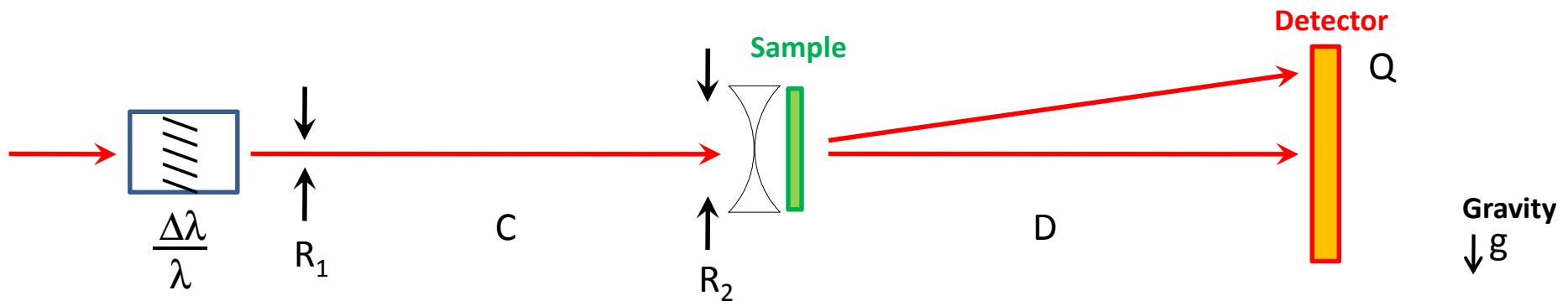
$$\sigma_Q^2 = a_1 \cdot R_1^2 + \left( \frac{\Delta\lambda}{\lambda} \right)^2 \left( a_2 \cdot R_2^2 + a_3 \cdot g^2 + a_4 \cdot Q^2 \right) + \text{crossterms}^4$$

↓      ↓      ↓  
small    negligible    for classical SANS

to Pedersen Resolution Function

$a_1, a_2, a_3, a_4$  (C,D, ...) determined by analytical calculations  
*and* computer simulations (McStas)  
done

# 1<sup>st</sup> step of resolution correction: isotropic, rad. averaged



## Abbreviations:

$$r_1 = \frac{2\pi}{\lambda D} \cdot \frac{D}{C} \cdot R_1$$

5E-4 .. 5E-5

$$r_2 = \frac{2\pi}{\lambda D} \cdot 2 \cdot \left(1 + \frac{D}{C}\right) \cdot R_2$$

9E-3

actually  
diameters !

$$q_0 = \frac{2\pi}{\lambda D} \cdot D \cdot (C + D) \cdot \frac{gm_n^2}{2h^2} \cdot \lambda^2$$

5E-4

Q  
1E-3 .. 1E-4

## Simulations with McStas

### Radial averaging of a Debye-Scherrer ring

$\Delta Q(r_1)$  from ( $r_1$  varied,  $r_2$  small, gravity off, fixed angle)

$\Delta Q(r_2)$  from ( $r_1$  small,  $r_2$  varied, gravity off, fixed angle)

$\Delta Q(g)$  from ( $r_1$  small,  $r_2$  small, gravity varied, fixed angle)

$\Delta Q(Q)$  from ( $r_1$  small,  $r_2$  small, gravity off, real scattering)

# Summary of resolution functions:

$$\sigma_Q^2 = 0.125 \cdot r_1^2 + \left( \frac{\Delta\lambda}{\lambda} \right)^2 (0.021 \cdot r_2^2 + 0.667 \cdot q_0^2 + 0.333 \cdot Q^2) \quad \text{Mildner}$$

$$\sigma_Q^2 = 0.250 \cdot r_1^2 + \left( \frac{\Delta\lambda}{\lambda} \right)^2 (0.016 \cdot r_2^2 + 0.472 \cdot q_0^2 + 0.236 \cdot Q^2) \quad \text{Analytical O(Q<sup>2</sup>)}$$

$$\sigma_Q^2 = 0.250 \cdot r_1^2 + \left( \frac{\Delta\lambda}{\lambda} \right)^2 (0.008 \cdot r_2^2 + 0.173 \cdot q_0^2 + 0.086 \cdot Q^2) \quad \text{Analytical O(Q<sup>4</sup>)}$$

$$\sigma_Q^2 = 0.107 \cdot r_1^2 + \left( \frac{\Delta\lambda}{\lambda} \right)^2 (0.026 \cdot r_2^2 + 0.096 \cdot q_0^2 + 0.242 \cdot Q^2) \quad \begin{matrix} \text{Simulations} \\ \text{McStas} \end{matrix}$$

$\Delta Q(r_1) = 1.6E-4$        $\Delta Q(r_2) = 1.5E-4$        $\Delta Q(g) = 1.5E-5$        $\Delta Q(Q) = 0.5E-4$       Classical SANS

$\Delta Q(r_1) = 1.6E-5$        $\Delta Q(r_2) = 1.5E-4$        $\Delta Q(g) = 1.5E-5$        $\Delta Q(Q) = 0.5E-5$       focussing SANS

2<sup>nd</sup> step of resolution correction: higher order terms

$$f\left(\begin{pmatrix} \delta q_x \\ \delta q_y \end{pmatrix}\right) = \exp\left[-\begin{pmatrix} \sigma_x^{-2} & \\ & \sigma_y^{-2} \end{pmatrix} \begin{pmatrix} \delta q_x^2 \\ \delta q_y^2 \end{pmatrix}\right] \cdot \left(1 + \begin{pmatrix} A_x \\ A_y \end{pmatrix} \begin{pmatrix} \delta q_x^2 \\ \delta q_y^2 \end{pmatrix} + \begin{pmatrix} \delta q_x^2 \\ \delta q_y^2 \end{pmatrix} \begin{pmatrix} B_{xx} & B_{xy} \\ B_{yx} & B_{yy} \end{pmatrix} \begin{pmatrix} \delta q_x^2 \\ \delta q_y^2 \end{pmatrix}\right)$$

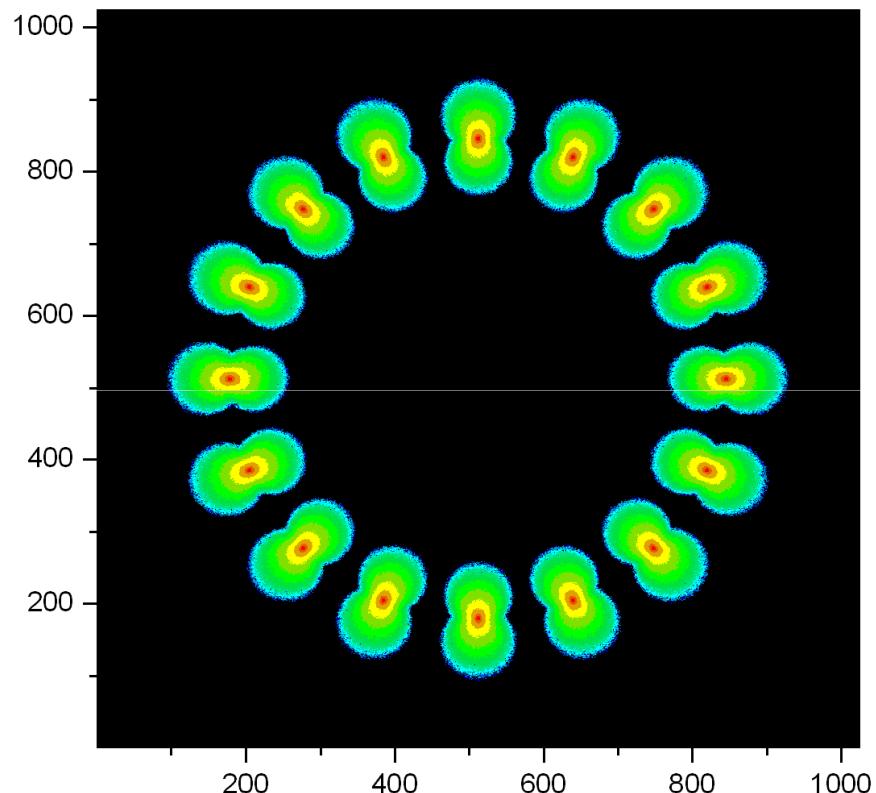
1<sup>st</sup> order resolution function  
(as before)      slight corrections      Peanut corrections

- a) Fit of smeared theoretical function
  - b) Desmearing of measured spectrum

## Current status:

Most important dependence of A,B ( $R_1, R_2, g, Q$ ) needs to be determined (analytically & from simulation)

No Gravity



With Gravity

