Simulating the μ^+SR experiment in high magnetic fields

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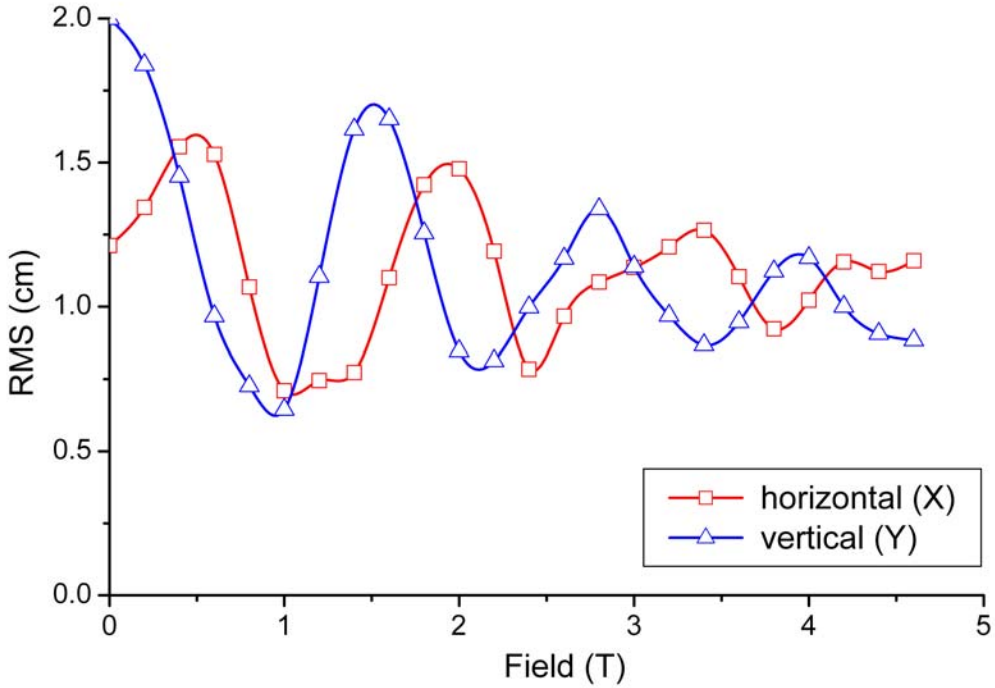
Introduction: why do we need instrument simulation?

How can we explain results like this



High fields are useful...

...but come with side-effects

Modify muon trajectory

changing beam-spot size at sample

Alter muon-spin polarization across sample

Modify positron trajectories

helical motion or "spiraling"

All alter the forward/backward asymmetry!

We need to understand these effects in detail

Simulations have been carried out using two separate program packages

Tofu and Geant4

The basic physics involved

The four vector $x^\mu = (ct, \mathbf{x})$

and the invariant $\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2$

The four velocity is $u^\mu = \dfrac{\mathrm{d}x^\mu}{\mathrm{d}s}$

The Maxwell field tensor

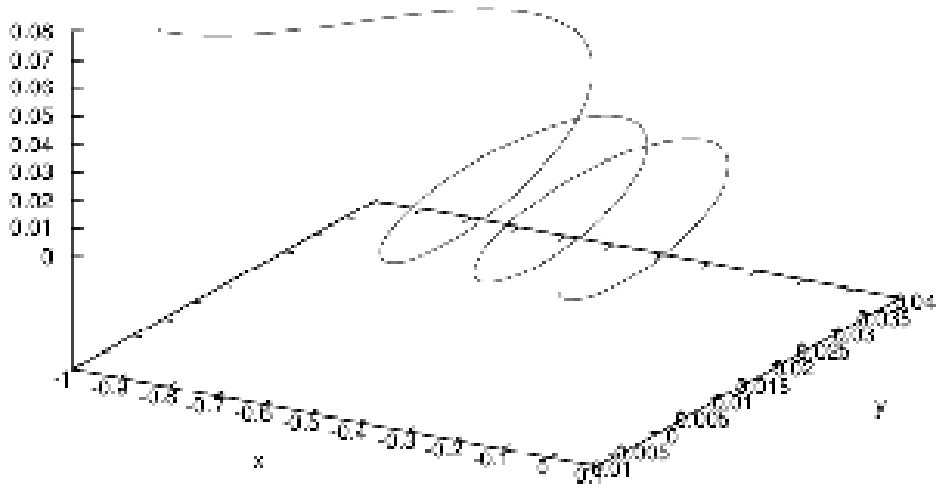
$$F^{ik} = \begin{pmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{pmatrix}.$$

Lead to the relativistic equations of motion

$$mc\frac{\mathrm{d}u^\mu}{\mathrm{d}s} = qF^{\mu\nu}u_\nu$$

But more importantly...

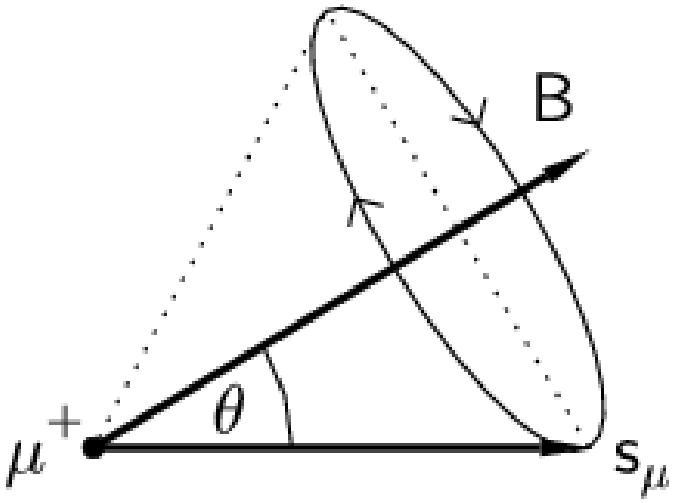
the cyclotron radius is given by $r = \dfrac{|\mathbf{v}_\mathrm{T}|\xi}{qBc^2}$



The basic physics involved ... continued

The equations of motion for the velocity

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \frac{e}{m}\mathbf{v} \times \mathbf{B}$$



are identical to those for the spin polarization

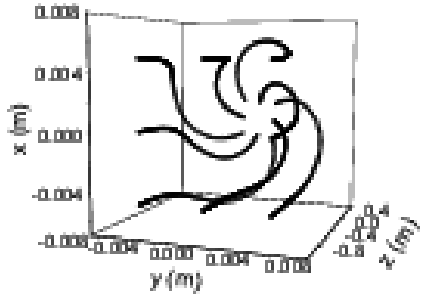
$$\frac{\mathrm{d}\mathbf{P}}{\mathrm{d}t} = \frac{e}{m}\mathbf{P} \times \mathbf{B}$$

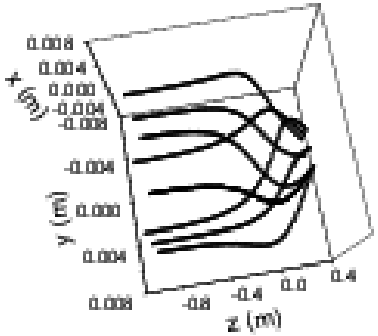
(even in the relativistic limit)

This provides a useful check for the calculations

(It's because g=2, *Quantum Electrodynamics*, Feynman 1962)

Tofu: A numerical approach





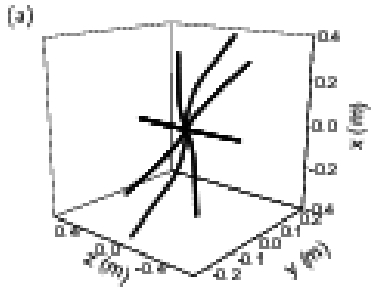
In-house routines

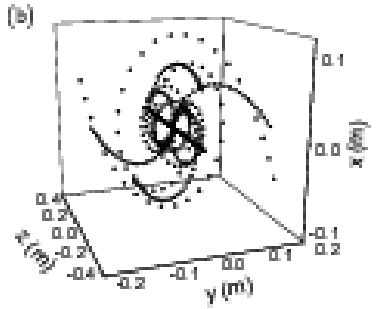
Written in FORTRAN

A quick and dirty method of obtaining results utilising Numerical Methods algorithms

Reads in field map $\longrightarrow$ calculates trajectories, evolves μ^+ spin

Includes relativistic correction, radiation damping





Implementing new geometries and physics is difficult...

...but the code's transparent and easily hacked!

T Lancaster, D.Phil Thesis, Oxford (2004)

Geant4: an object oriented approach

Powerful toolkit used for particle simulations

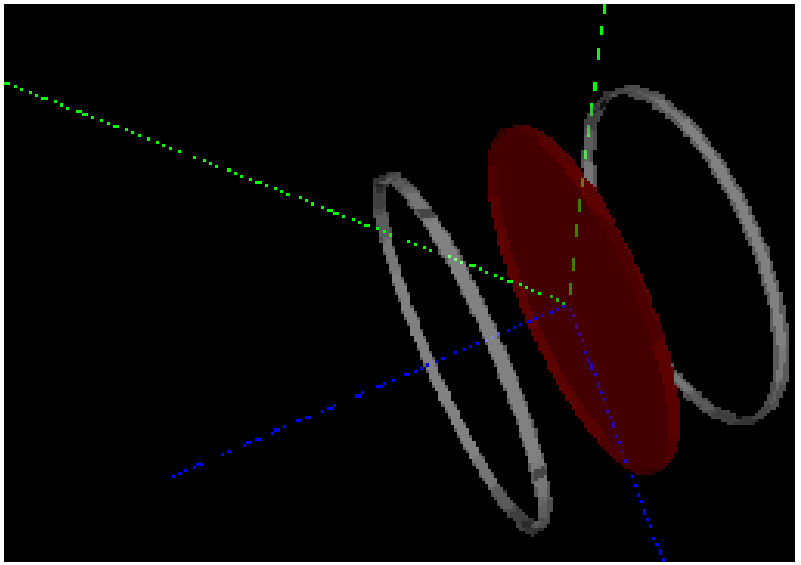
Written in C++, exploiting software engineering and object oriented technology

Includes tracking, geometry, physics models and hits

Able to cope with complex geometries and complex
interplay of physical processes

Extensive user support and documentation

Primarily used by HEP community,
steep learning curve
and the code's rather daunting!



http://wwwasd.web.cern.ch/wwwasd/geant4/geant4.html

Initial conditions for simulations

Principle field direction along z, with cylindrical symmetry

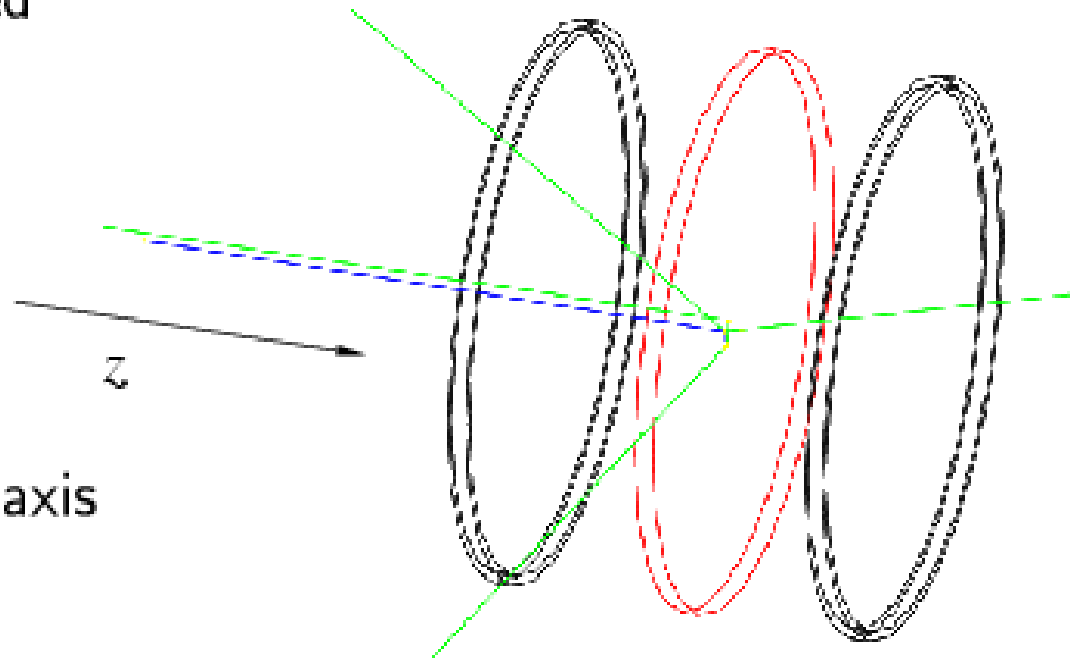
Muons start at -z_0=1 m with ξ_{kin}=4.12 MeV

Momentum directed along $+z$

Polarization $\mathbf{P}$ initially along either -z (longitudinal) or x (transverse)

Muons stoped in silver at z=0, where their position x^μ and polarization $\mathbf{P}$ is recorded

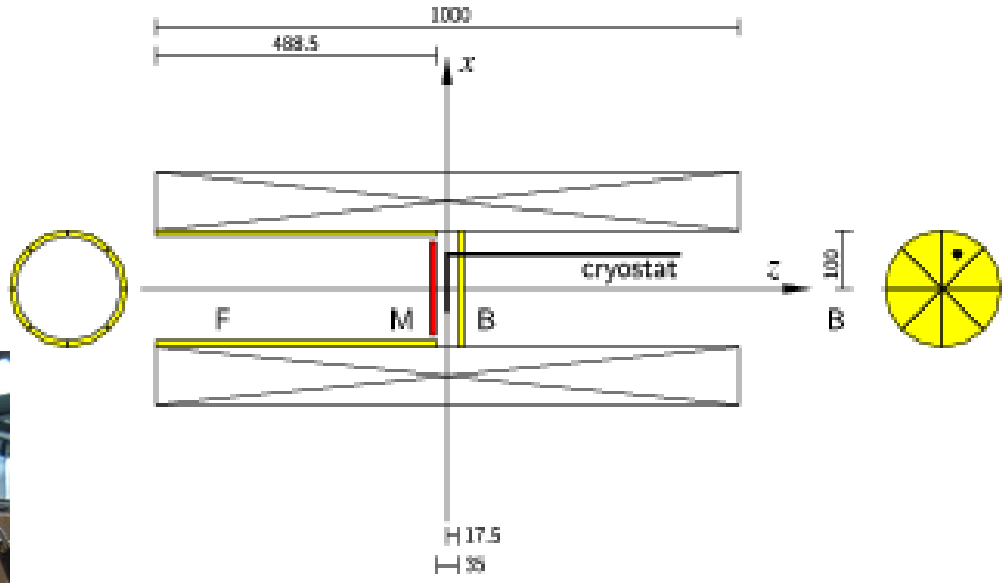
We'll consider a beam with a circular profile, and individual muons at distance r from the z-axis

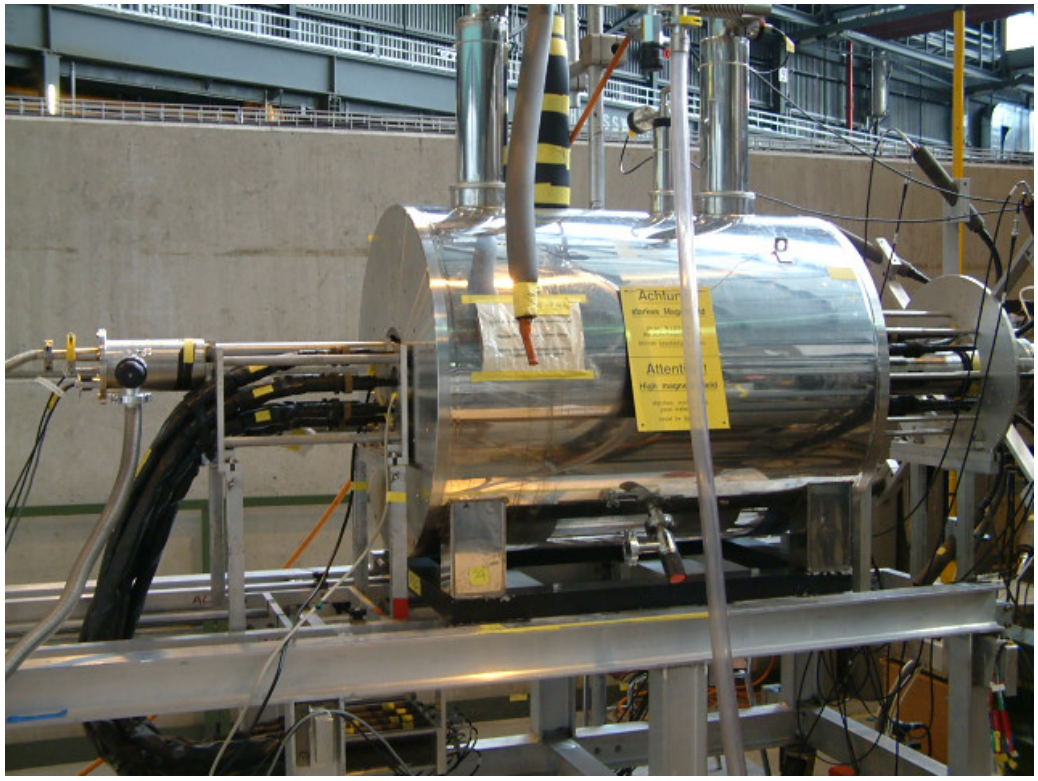


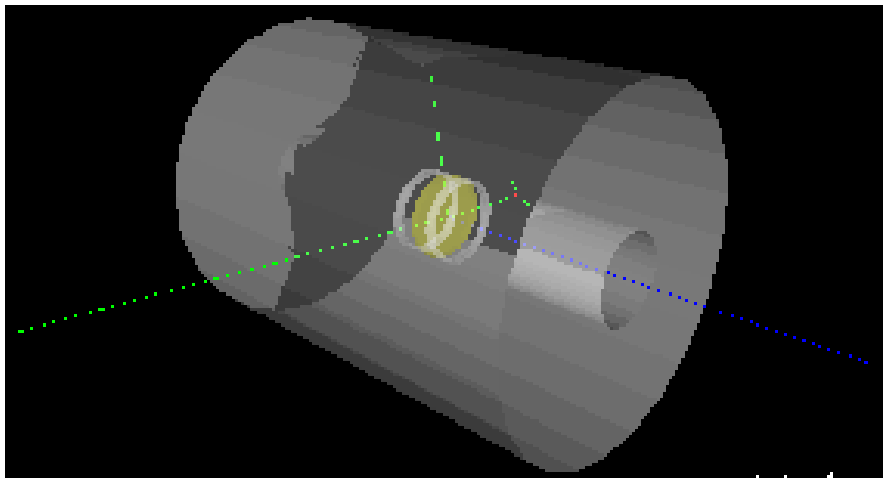
Simulation of the ALC spectrometer

Field calculated using Biot-Savart law

$$\mathbf{dB}(\mathbf{x}) = \frac{\mu_0 I \mathbf{dl} \times (\mathbf{x} - \mathbf{x}')}{4\pi |\mathbf{x} - \mathbf{x}'|^3}$$

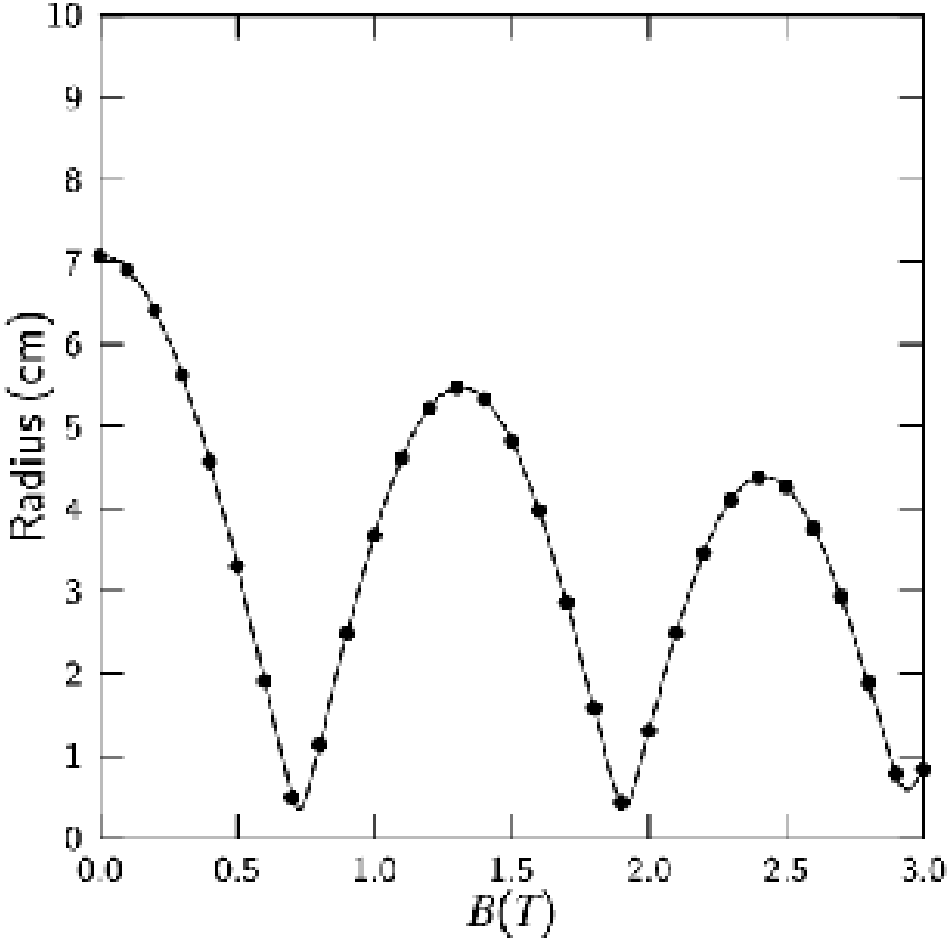




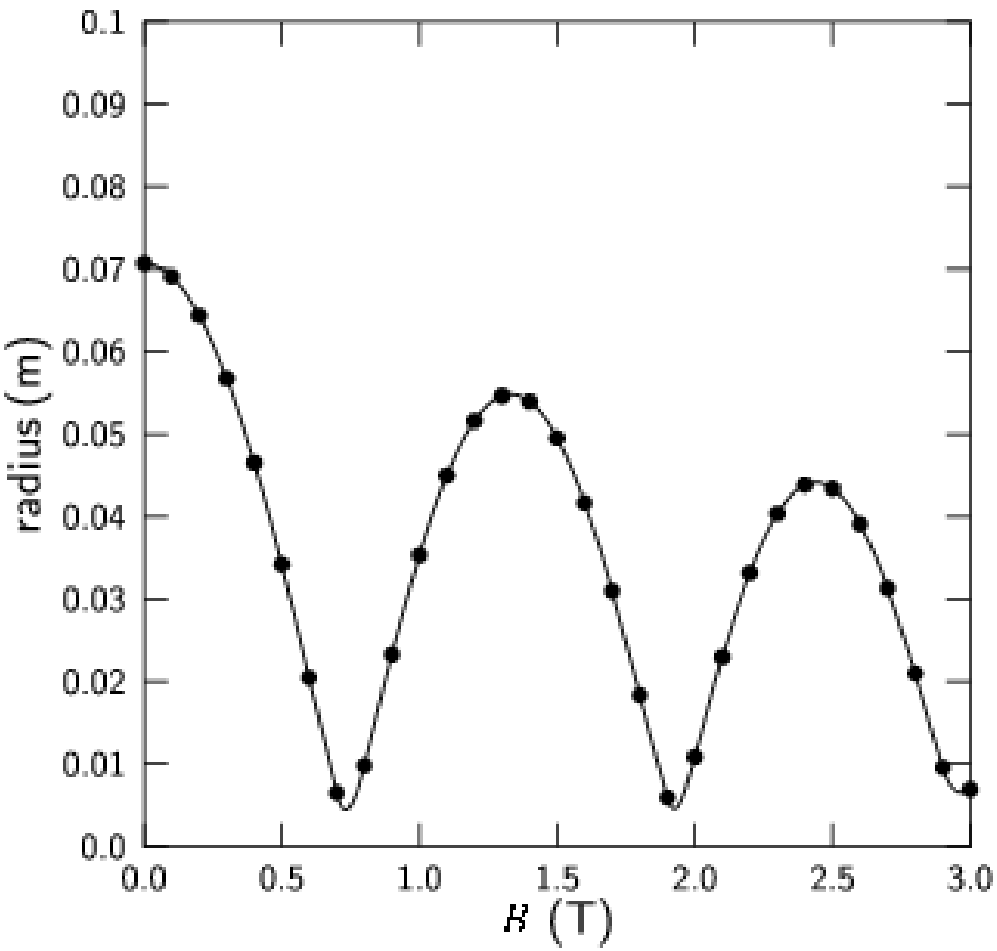


ALC: Pulsing of the spot size

Geant4



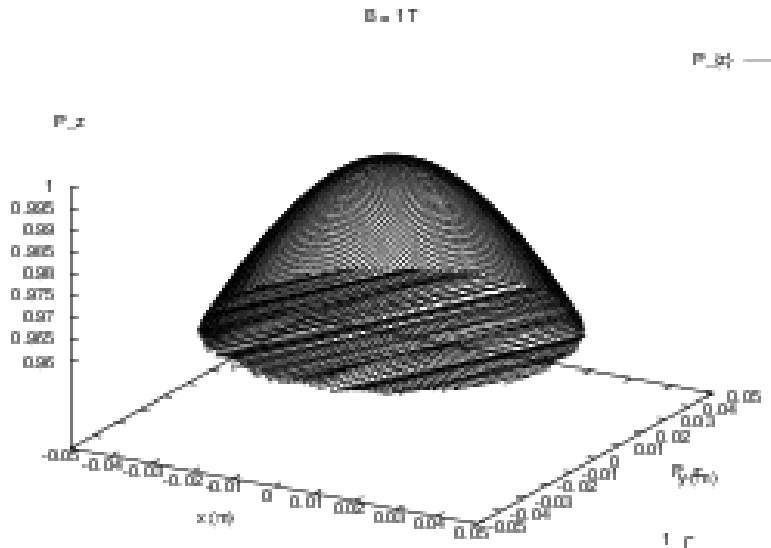
Tofu

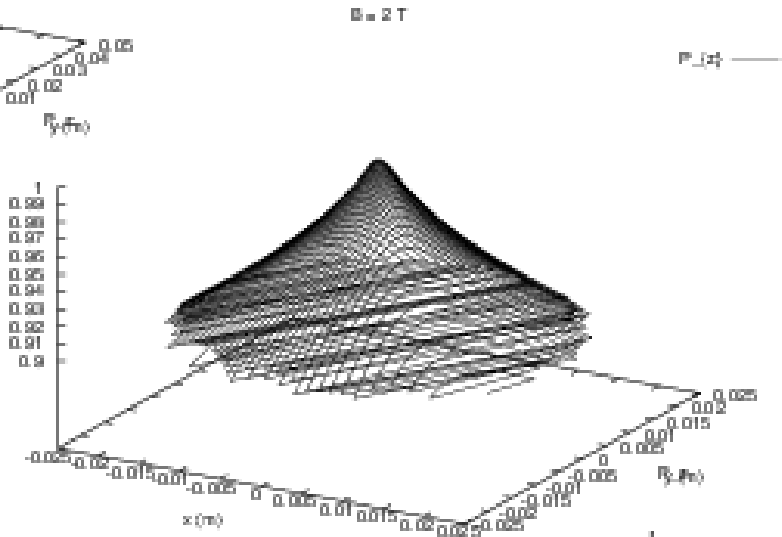


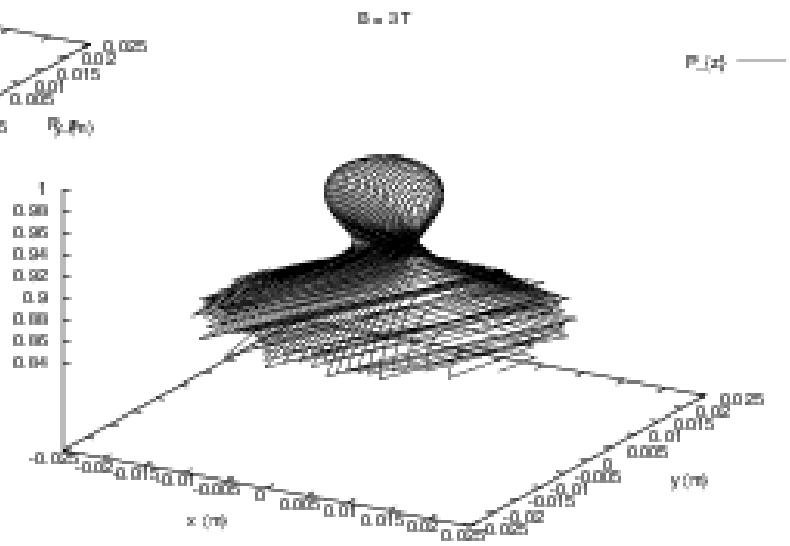
Good agreement!

ALC: Phase distribution

Polarization initially along z

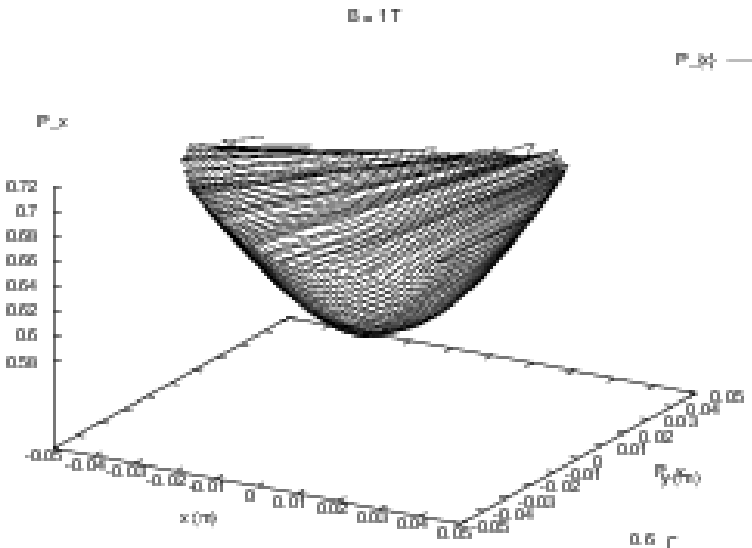


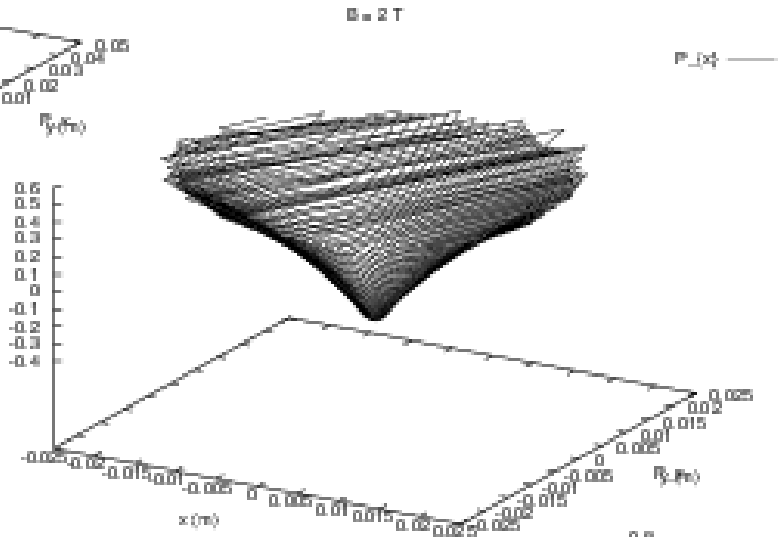


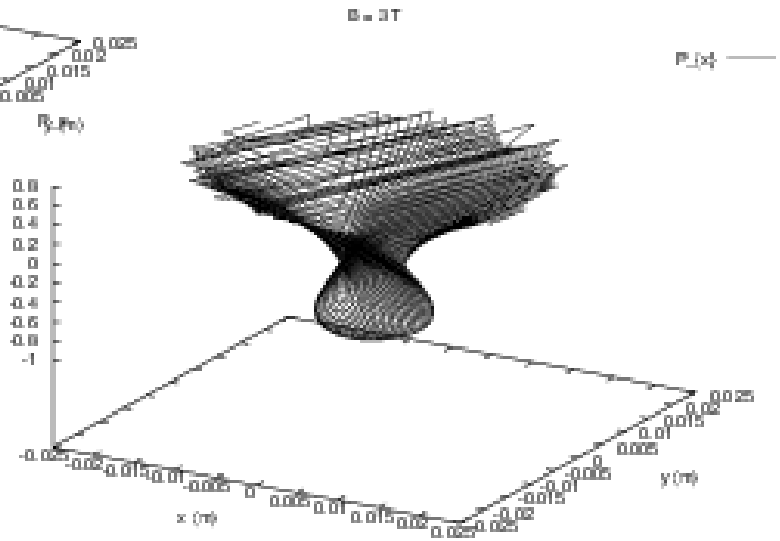


ALC: Phase distribution

Polarization initially along x





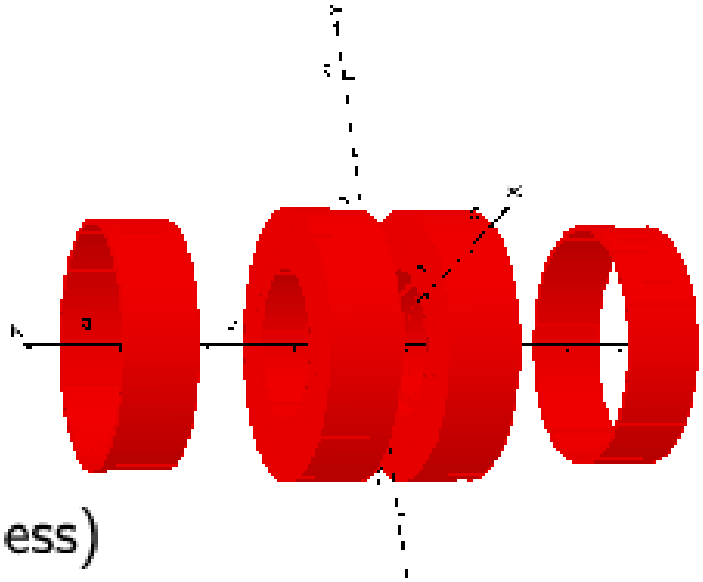


Simulations using the proposed magnet

Design study from Cryogenic Ltd.

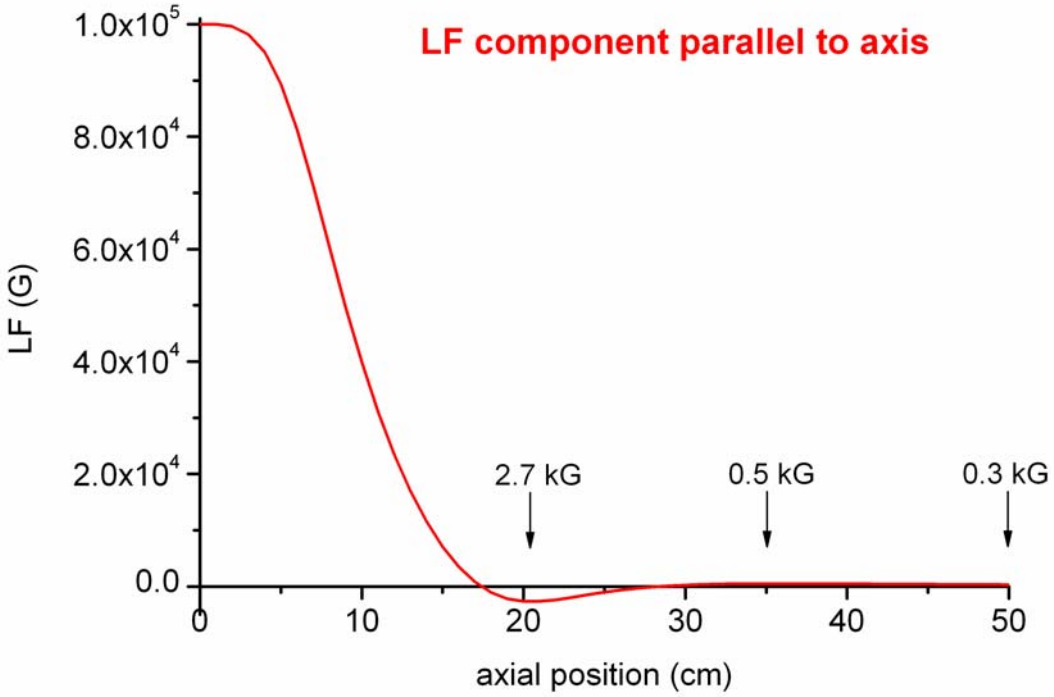
axial bore diameter = 68 mm

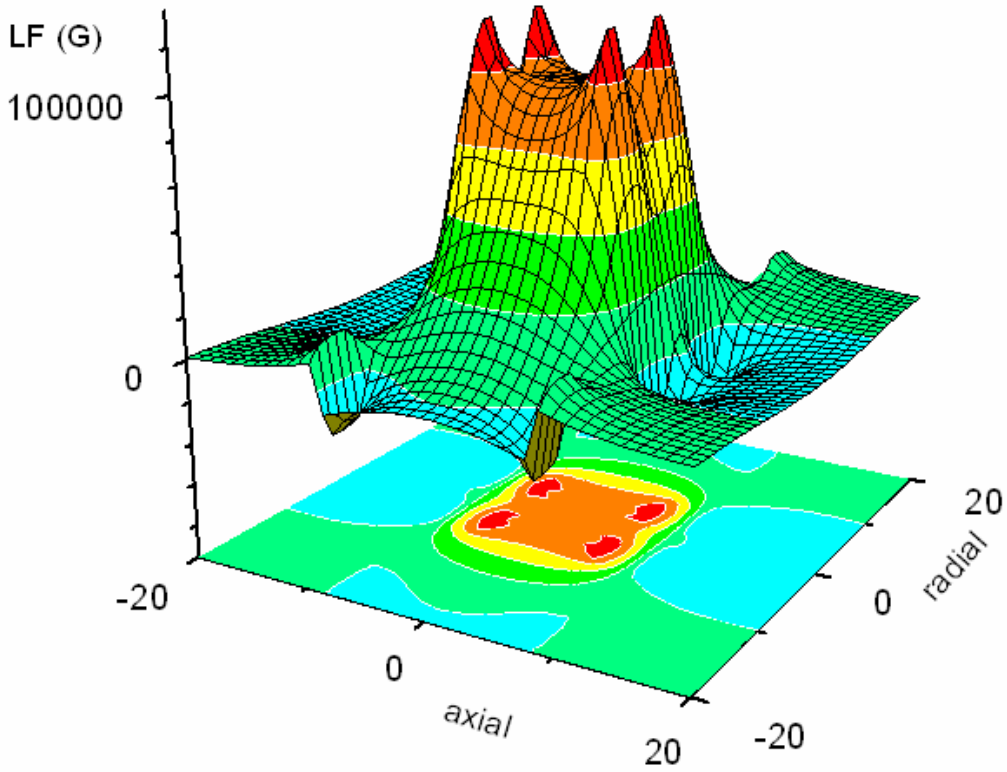
coil split 28 mm



Homogeneity 10.28 ppm
(over a disc 10 mm diameter, 5 mm thickness)

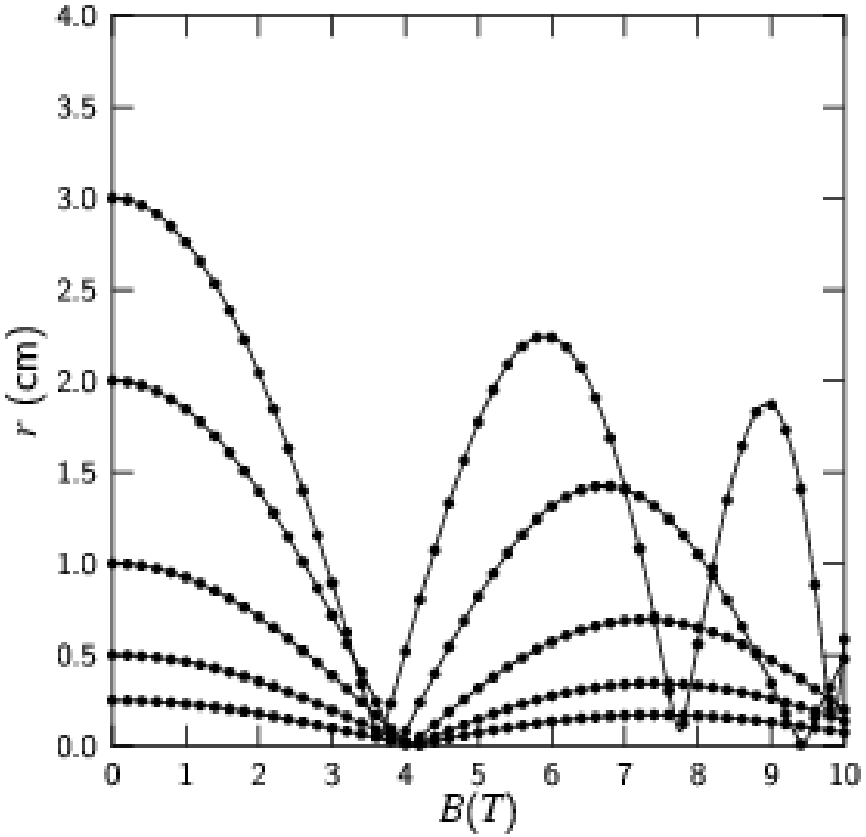
'Real' field maps provided



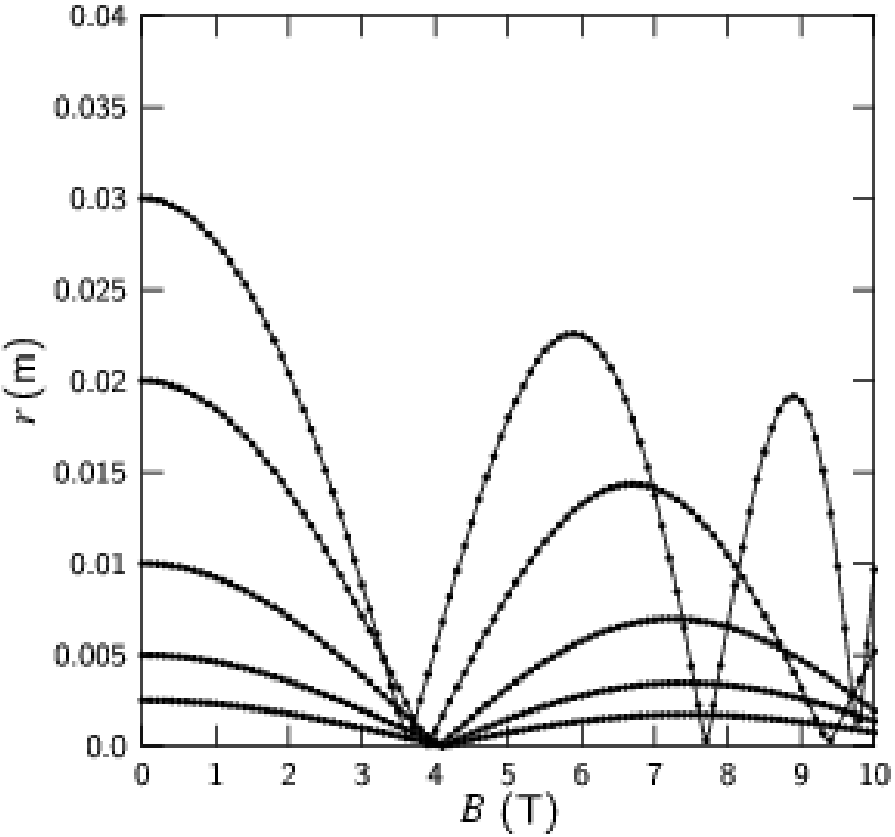


Cryogenic magnet: Pulsing of the spot size

Geant4



Tofu

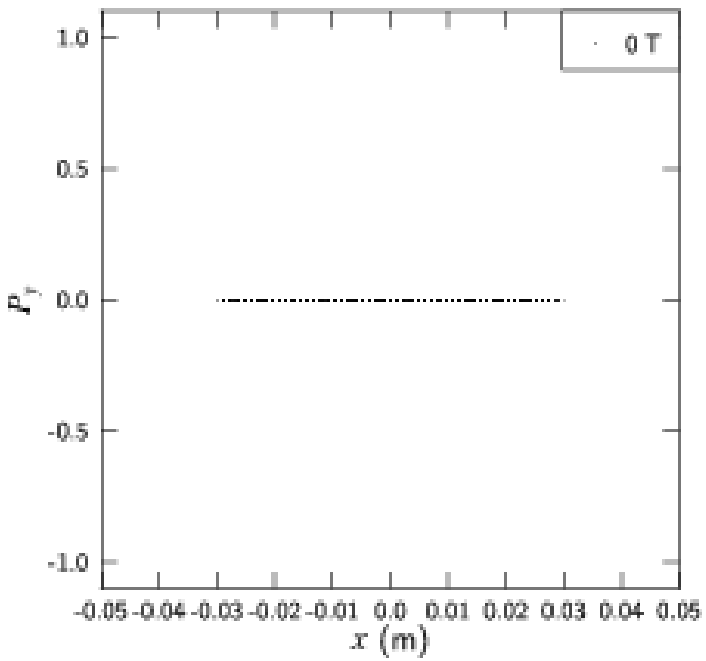


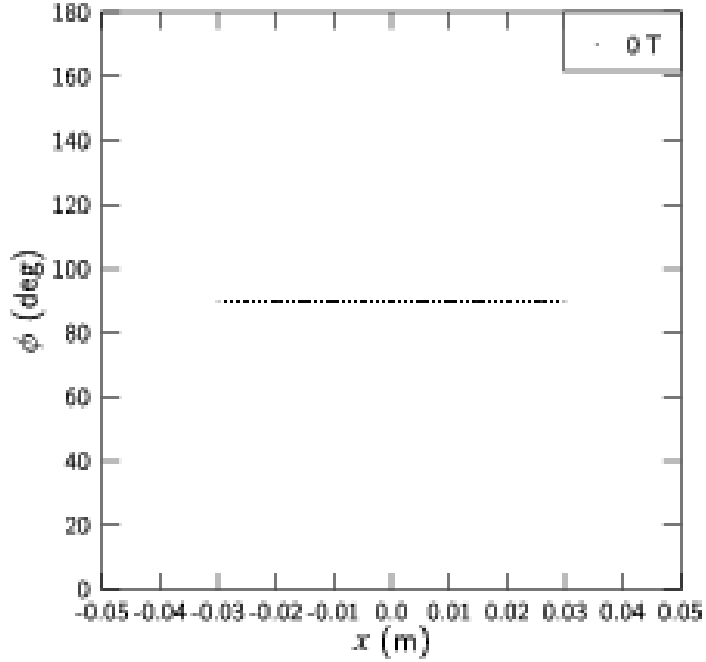
Good agreement!

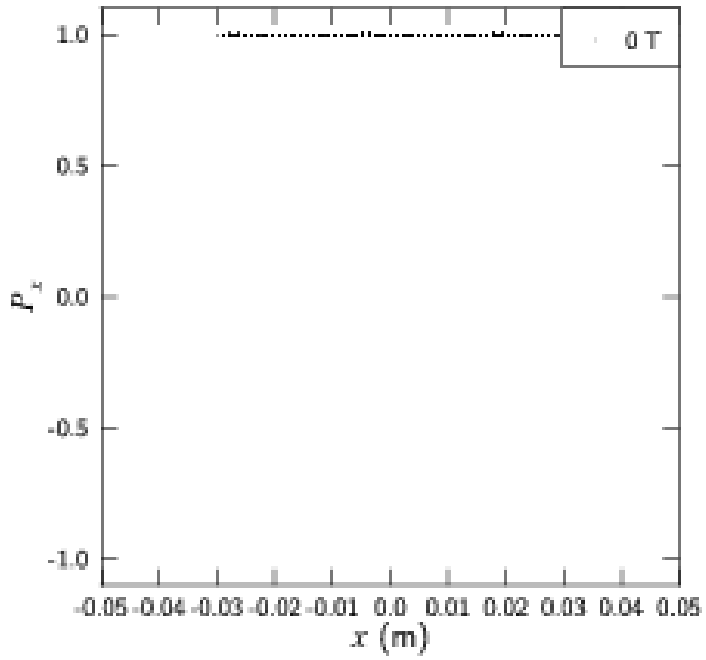
Polarization initially along x

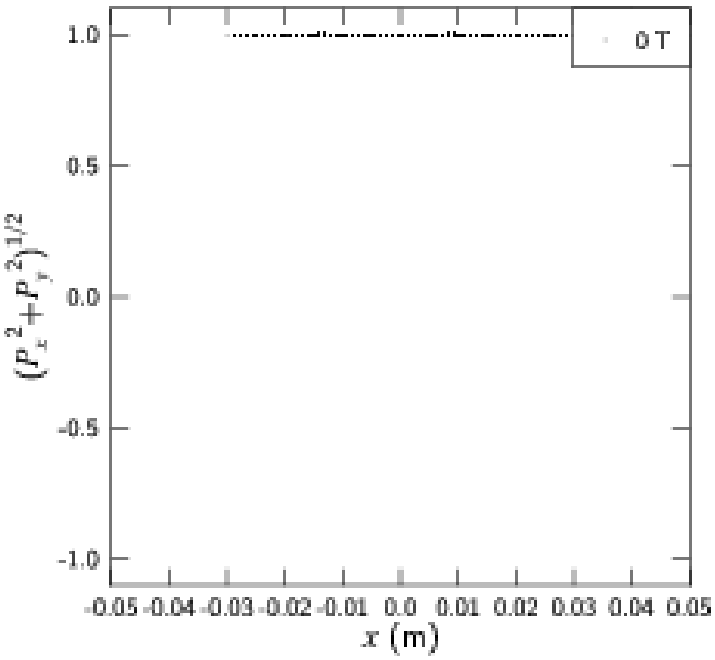
Beam radius 3 cm

$B = 0$ T





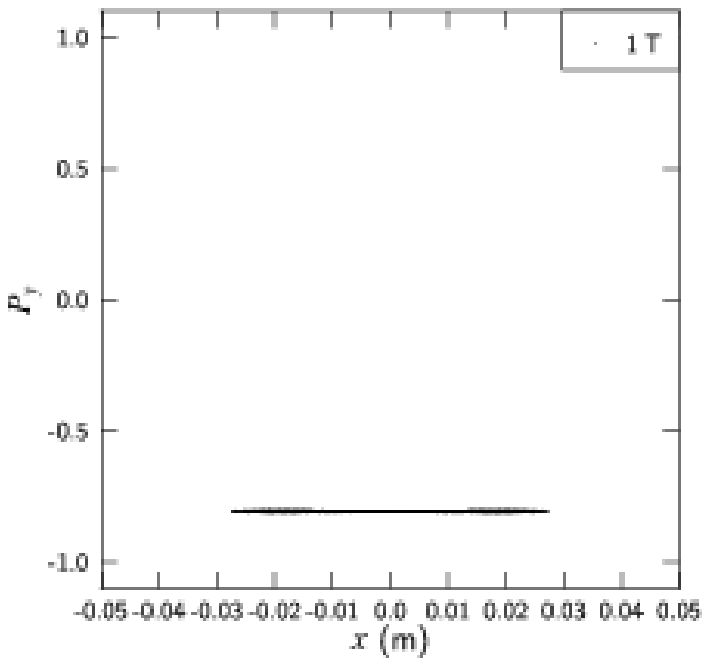


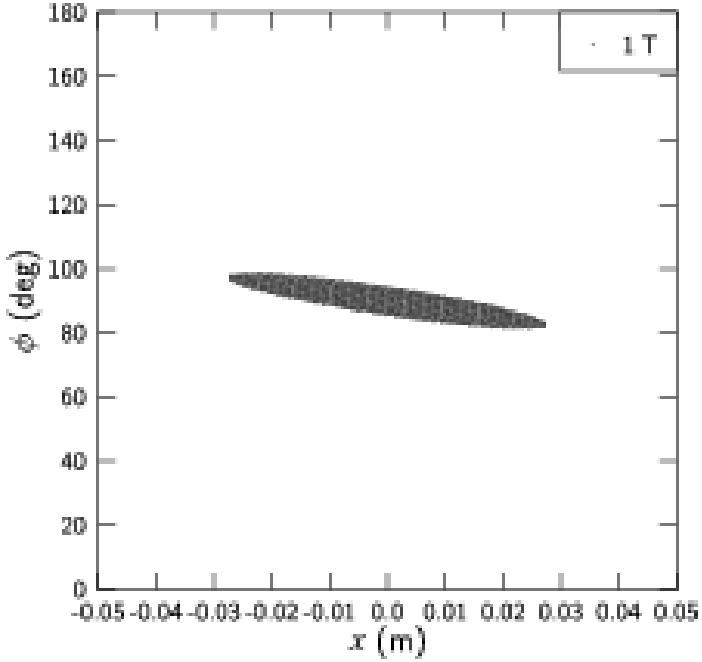


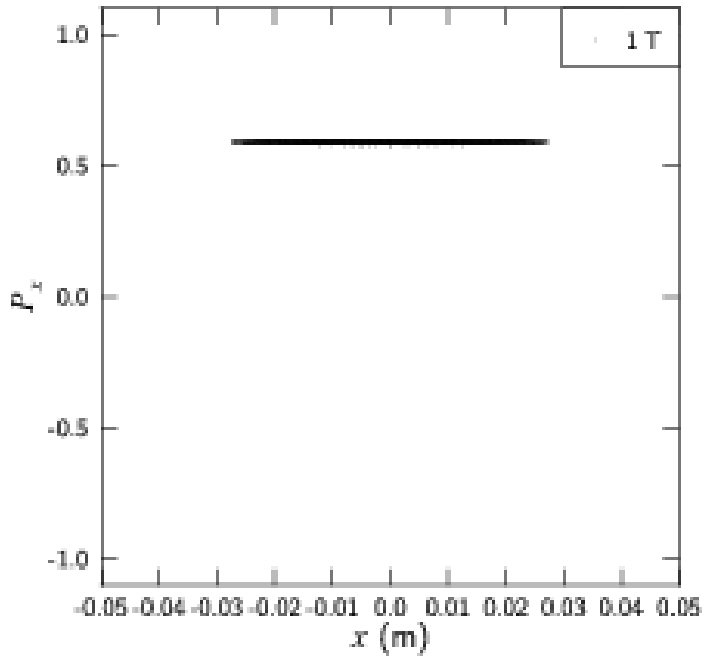
Polarization initially along x

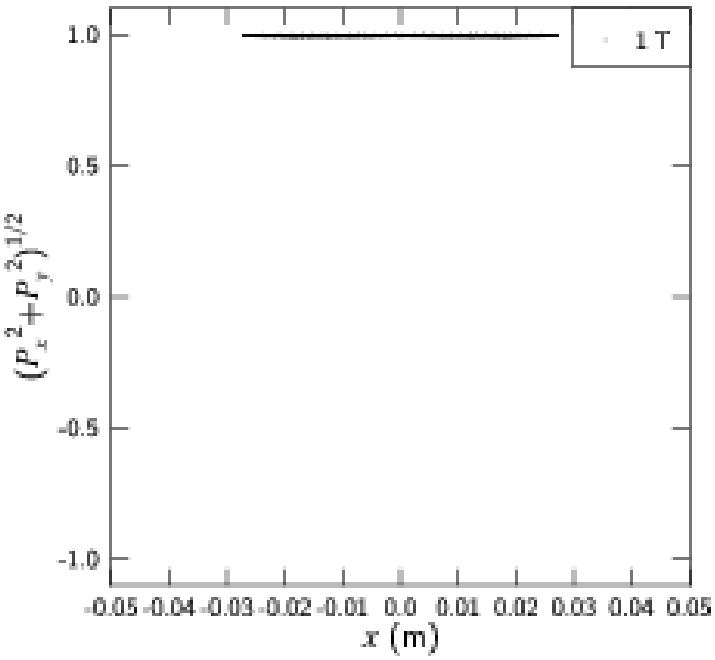
Beam radius 3 cm

$B = 1$ T





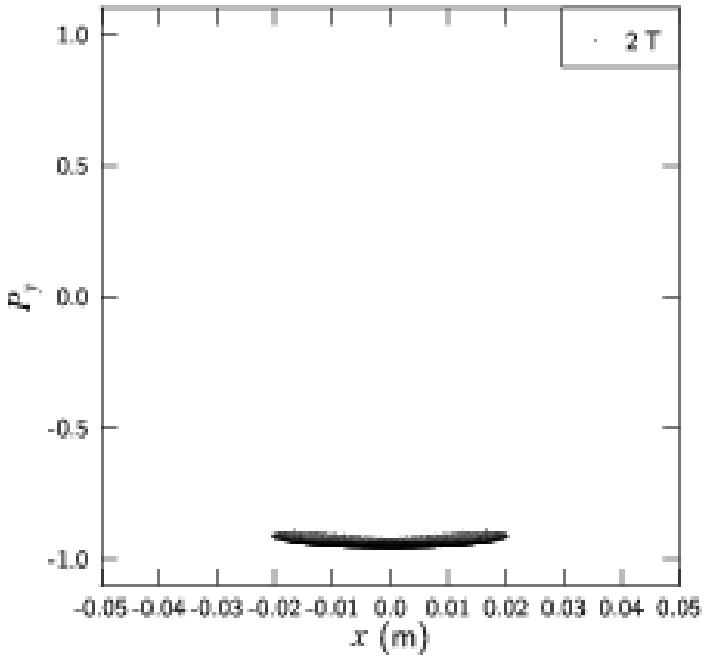


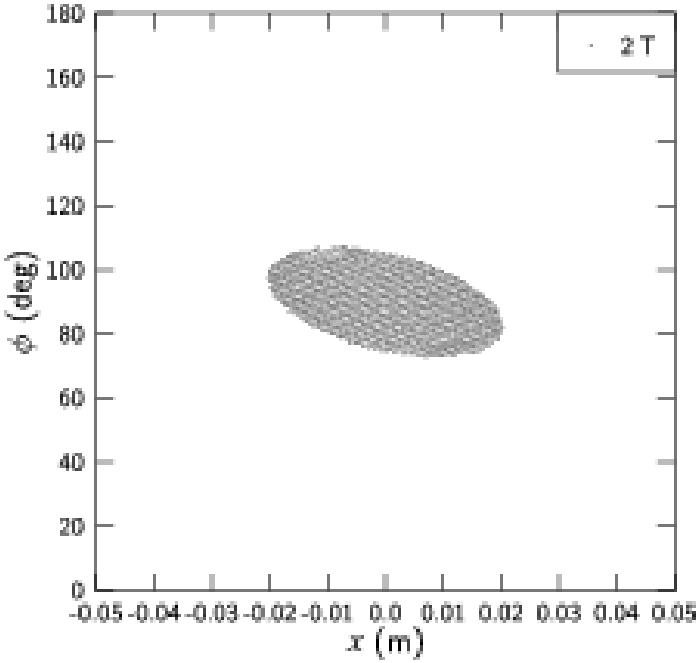


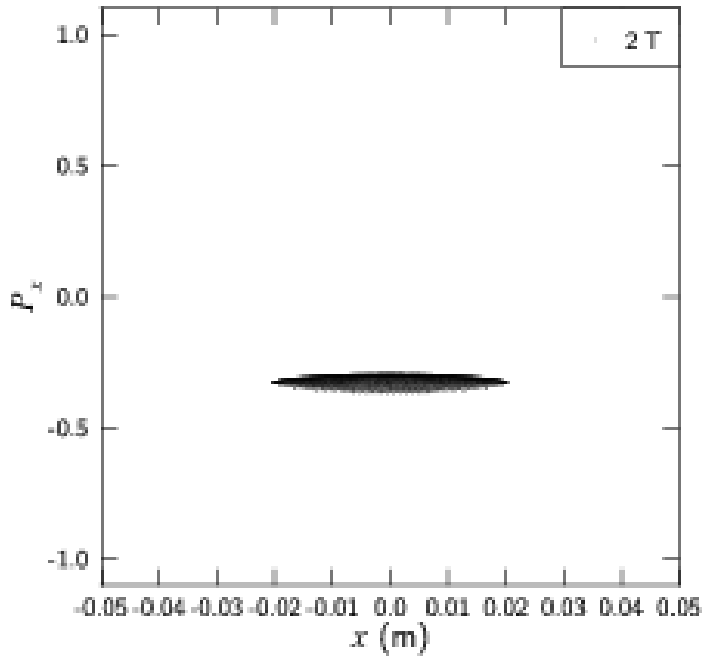
Polarization initially along x

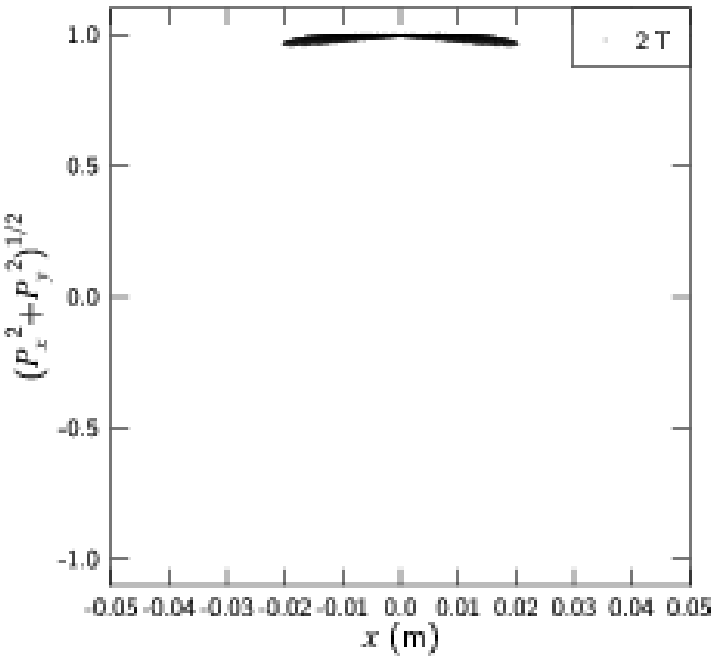
Beam radius 3 cm

$B = 2$ T





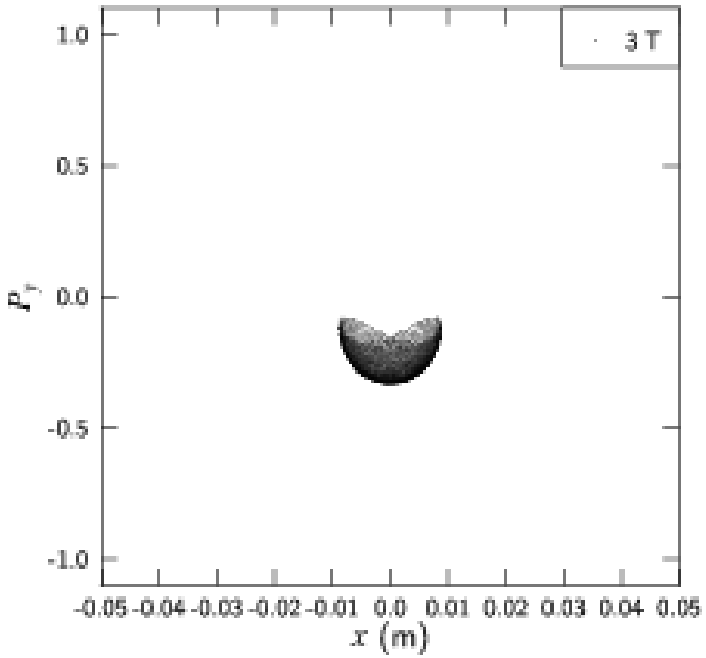


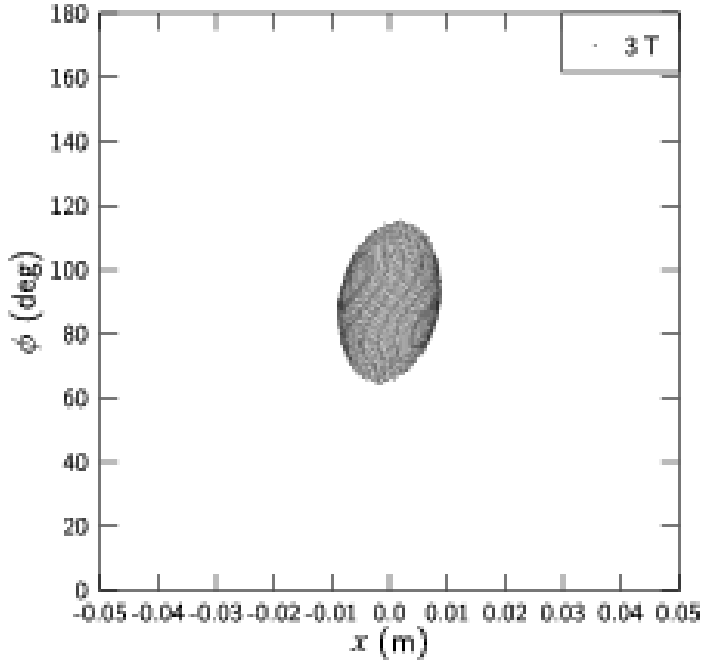


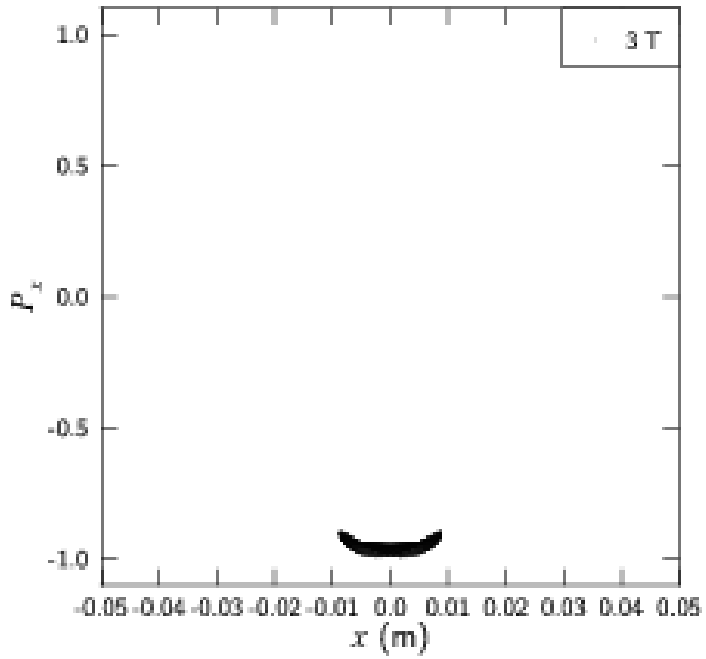
Polarization initially along x

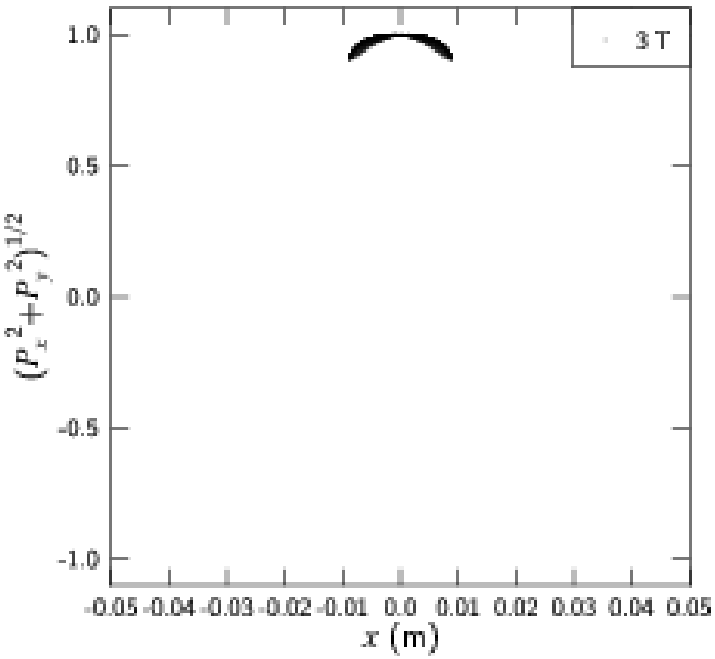
Beam radius 3 cm

$B = 3$ T





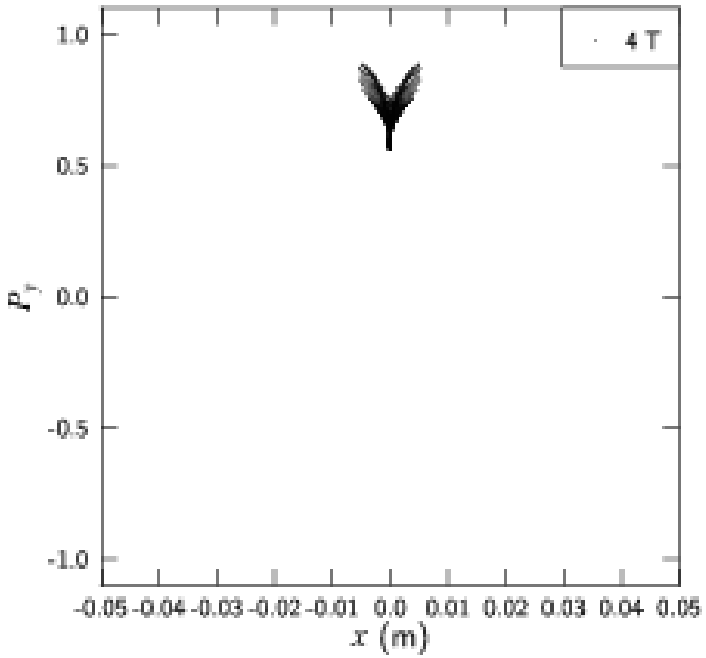


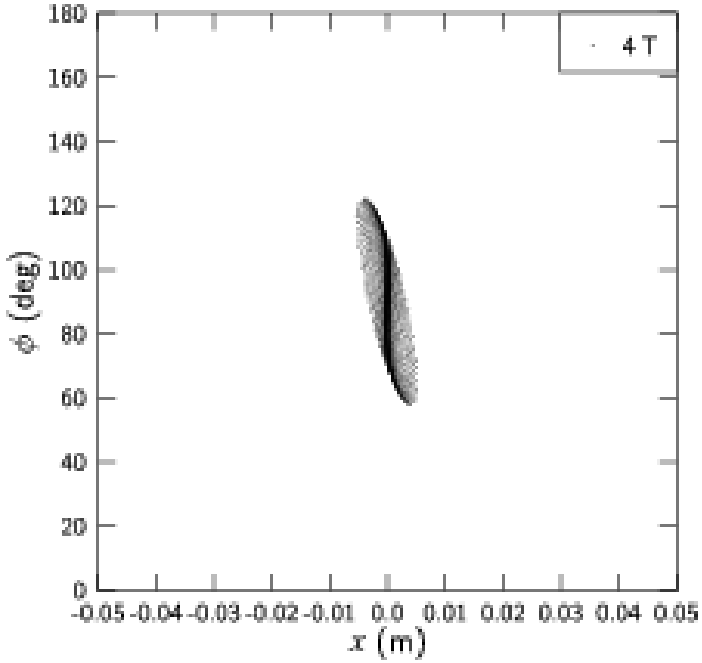


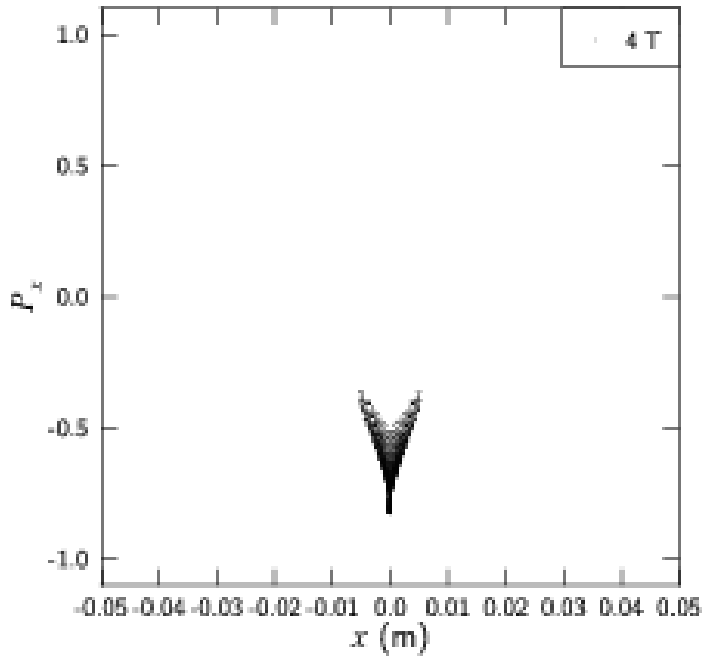
Polarization initially along x

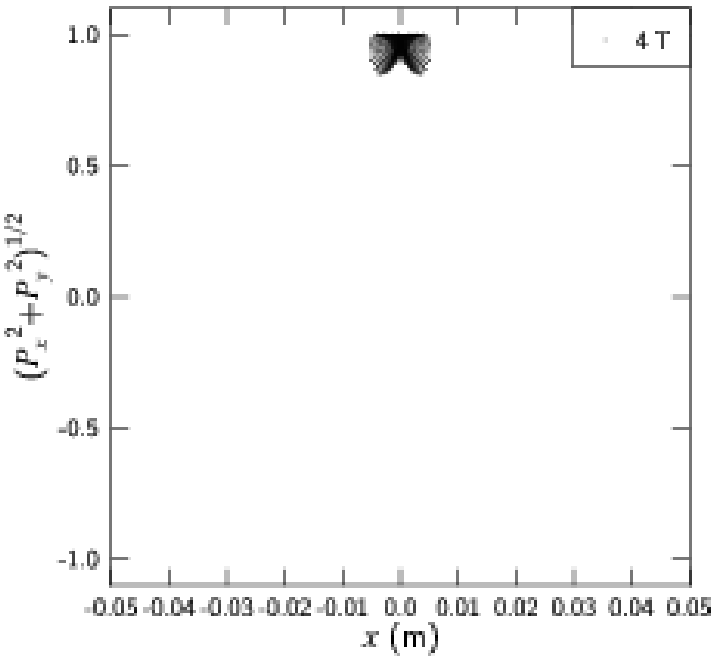
Beam radius 3 cm

$B = 4$ T





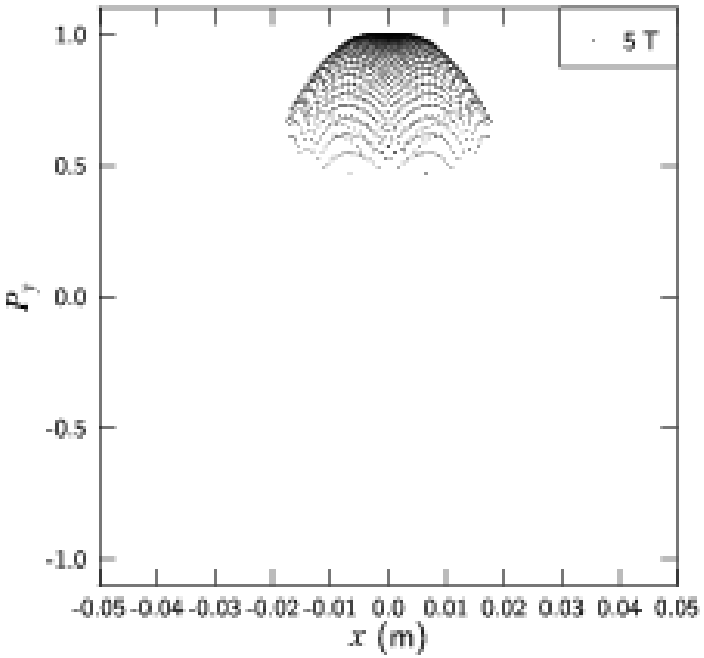


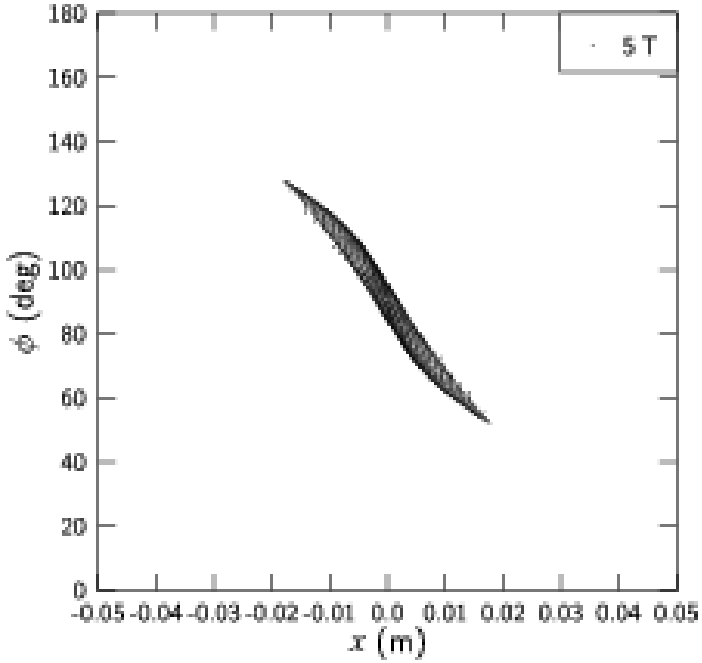


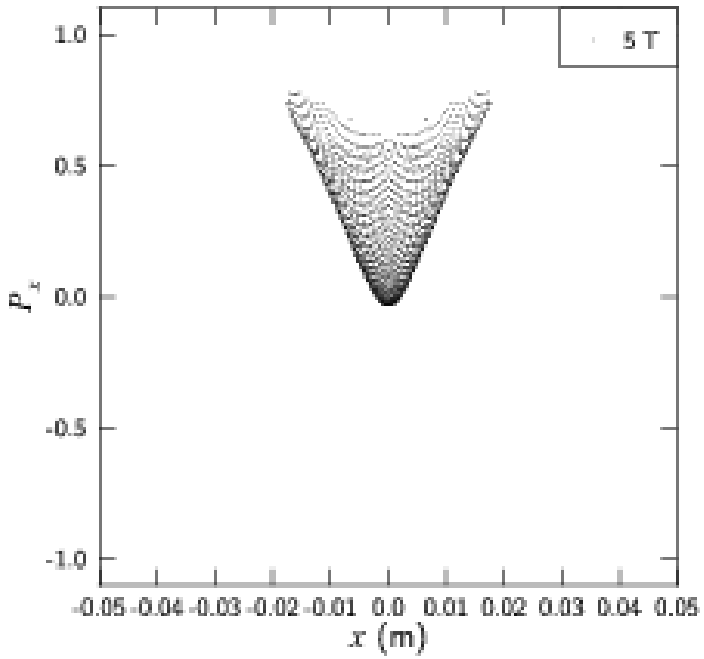
Polarization initially along x

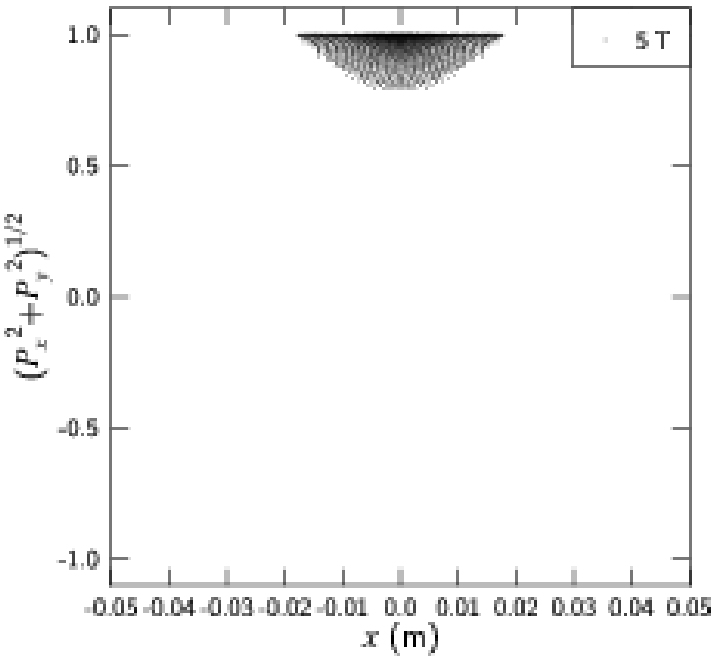
Beam radius 3 cm

$B = 5$ T





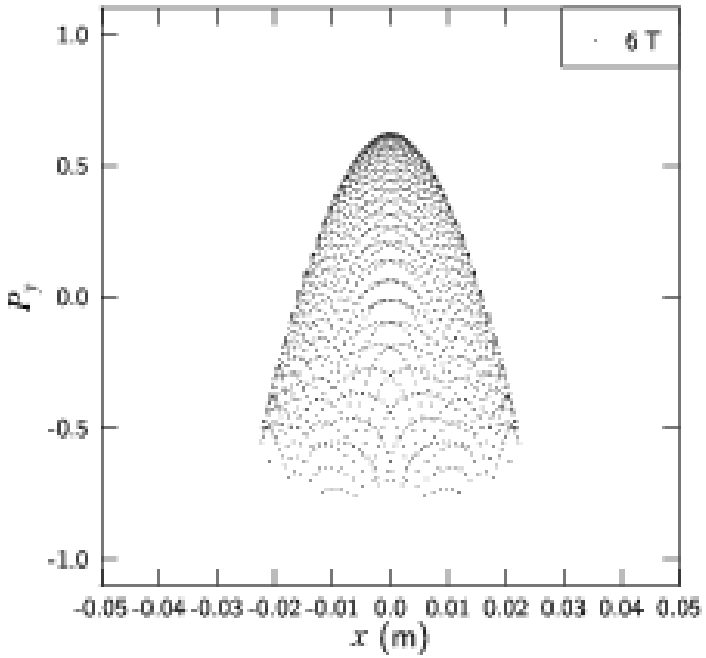


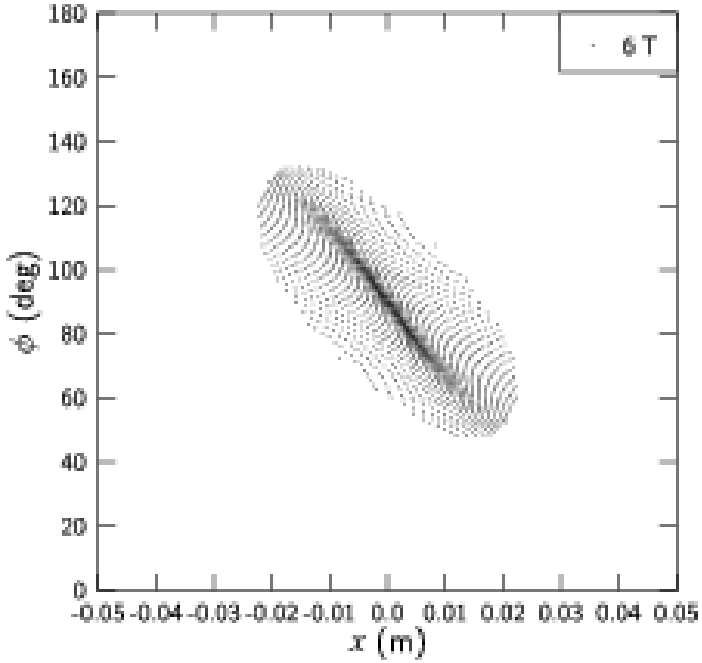


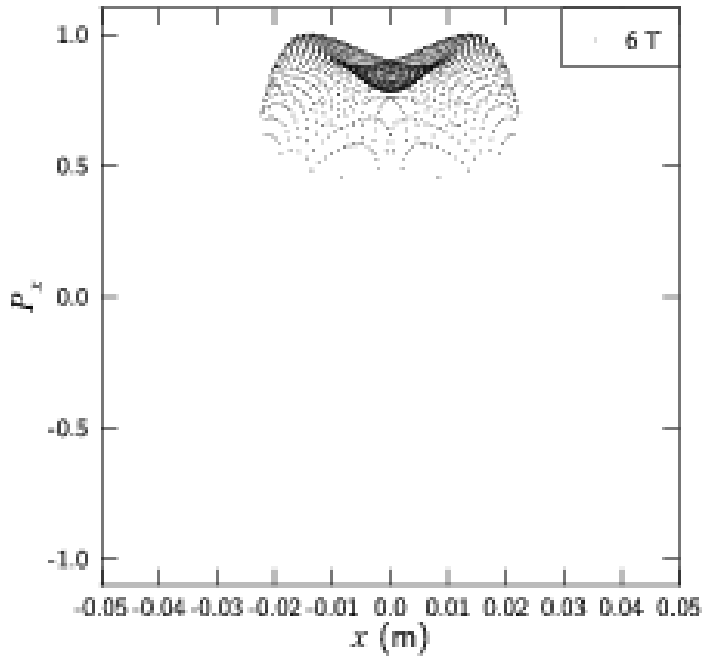
Polarization initially along x

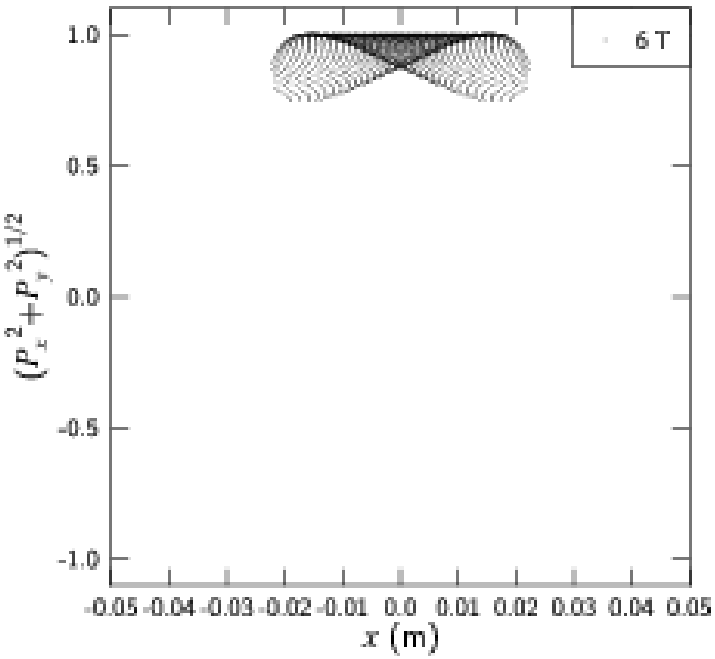
Beam radius 3 cm

$B = 6$ T





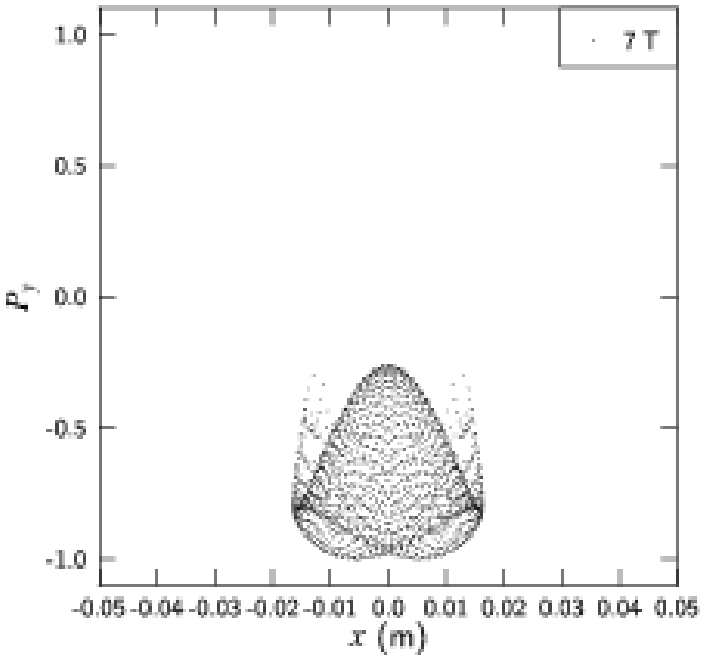


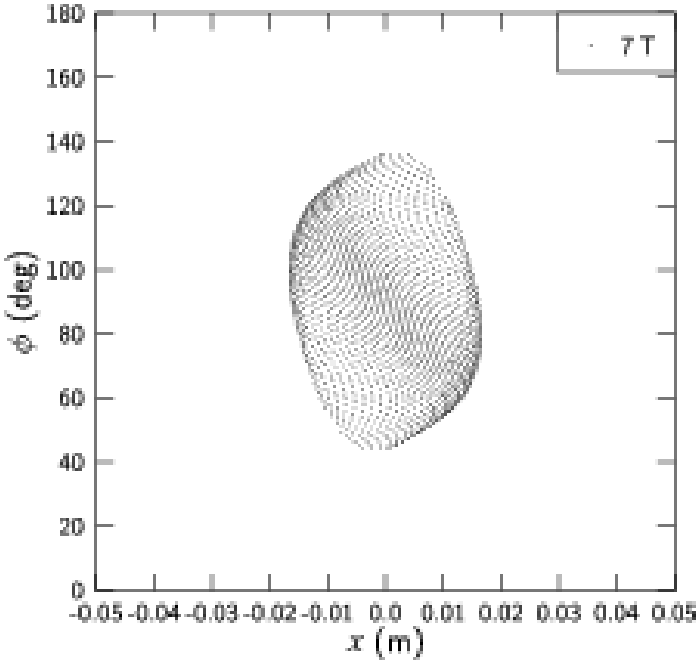


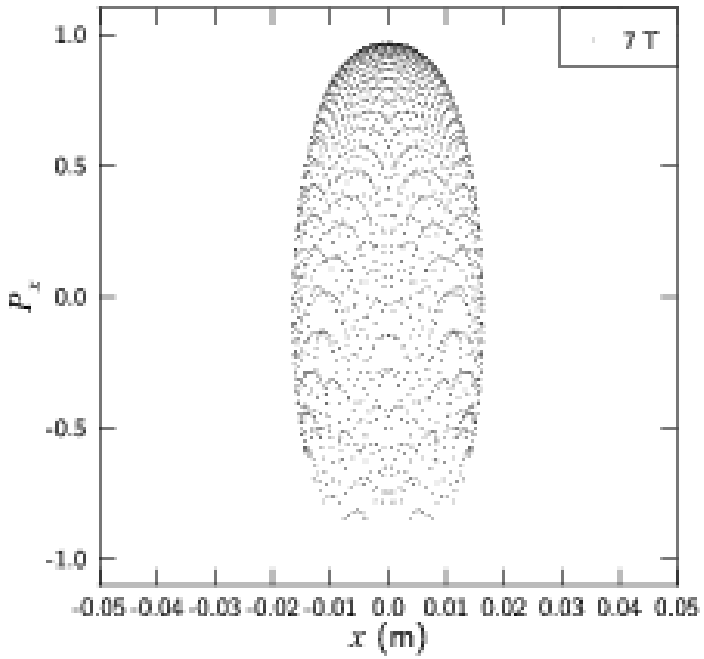
Polarization initially along x

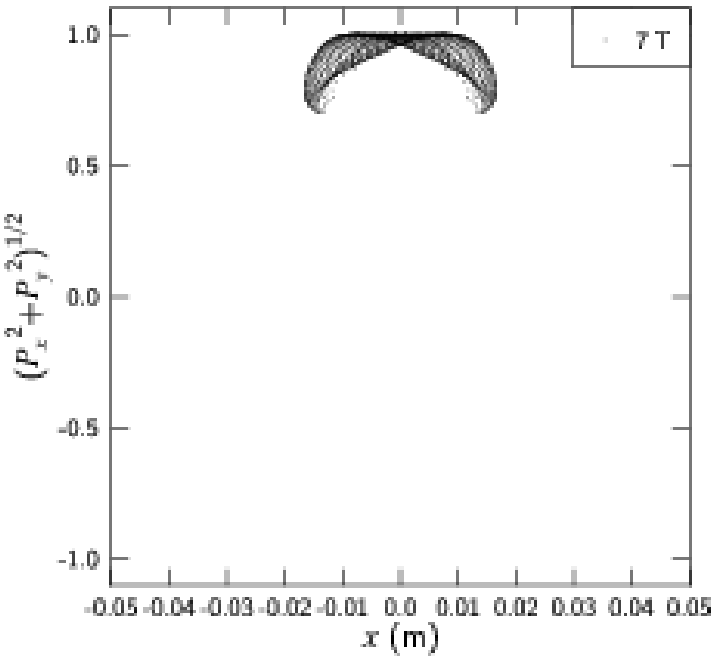
Beam radius 3 cm

$B = 7$ T





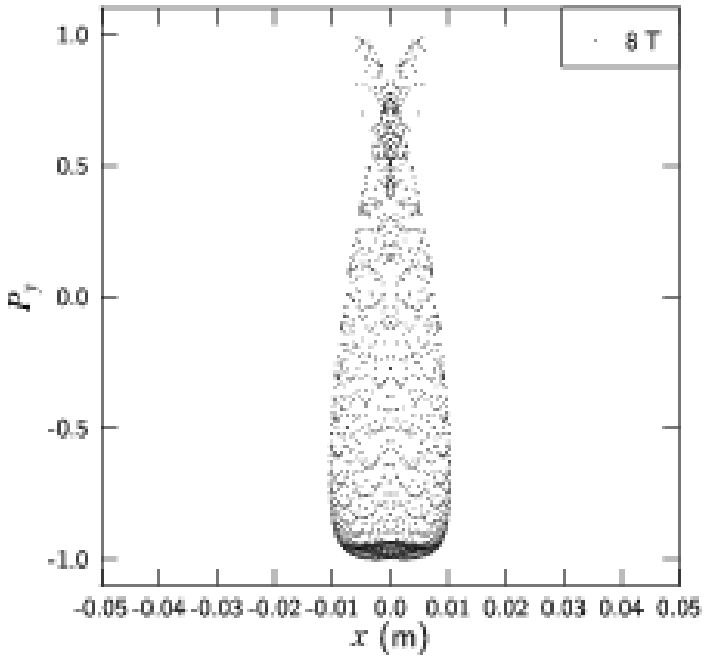


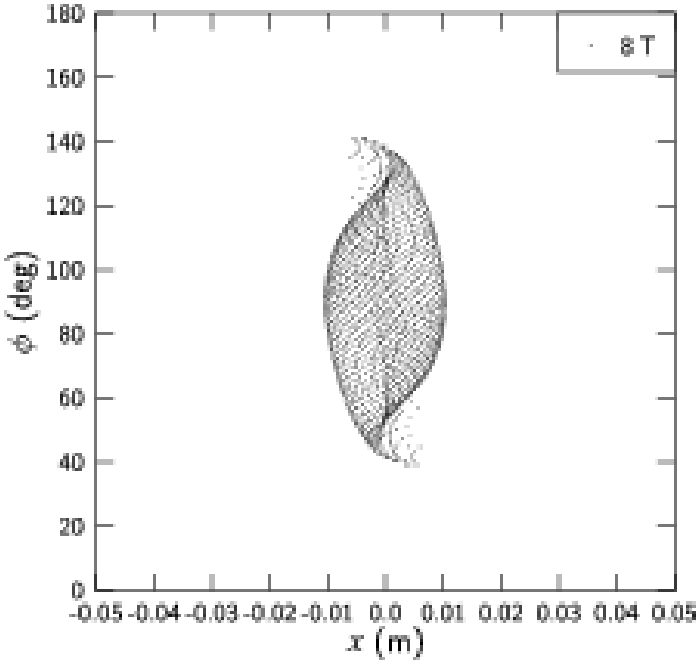


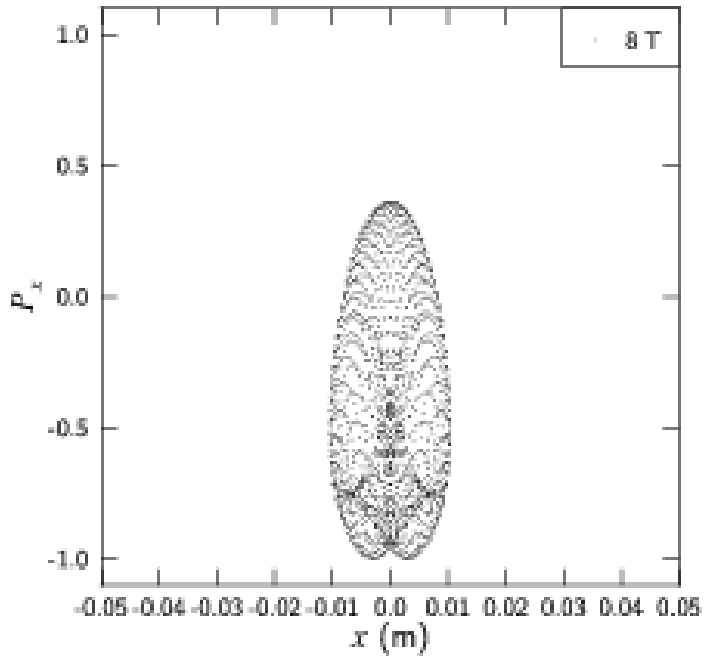
Polarization initially along x

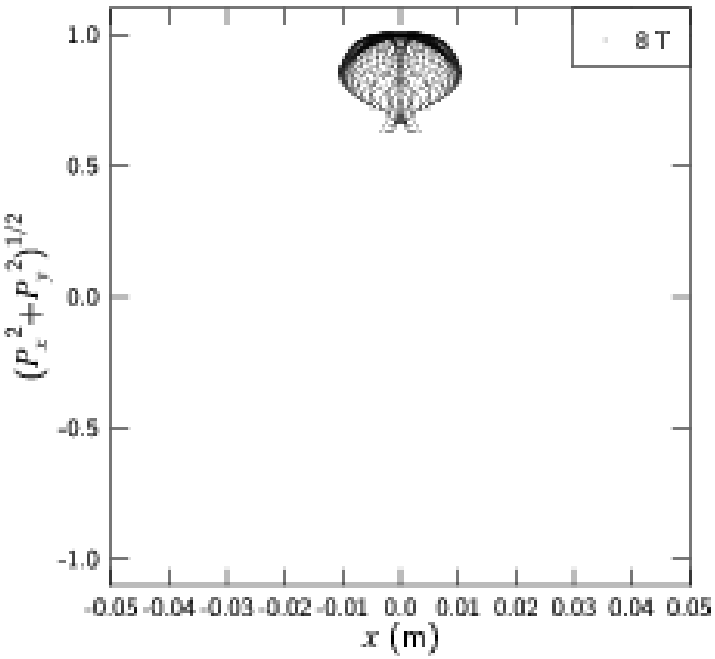
Beam radius 3 cm

$B = 8$ T





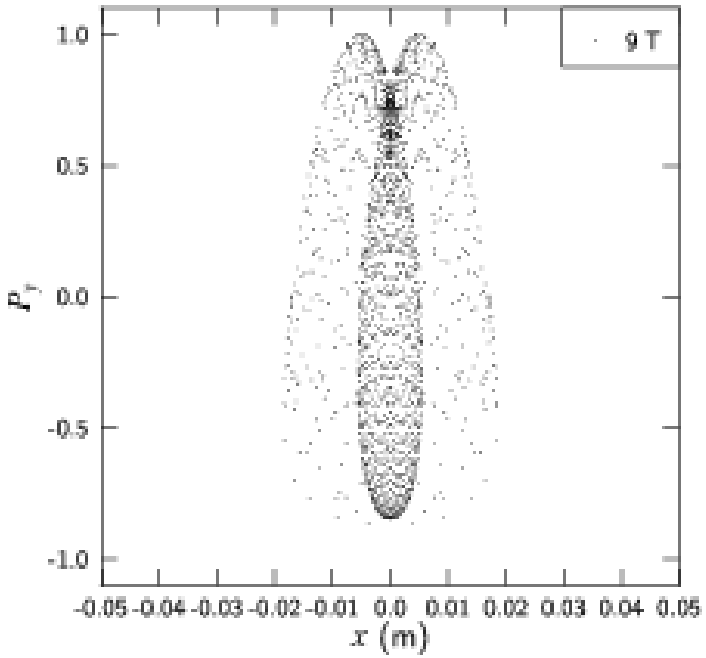


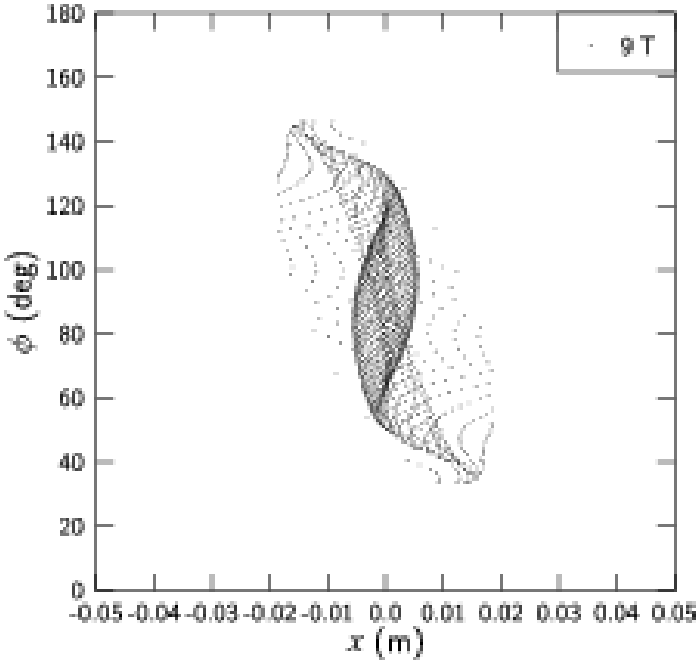


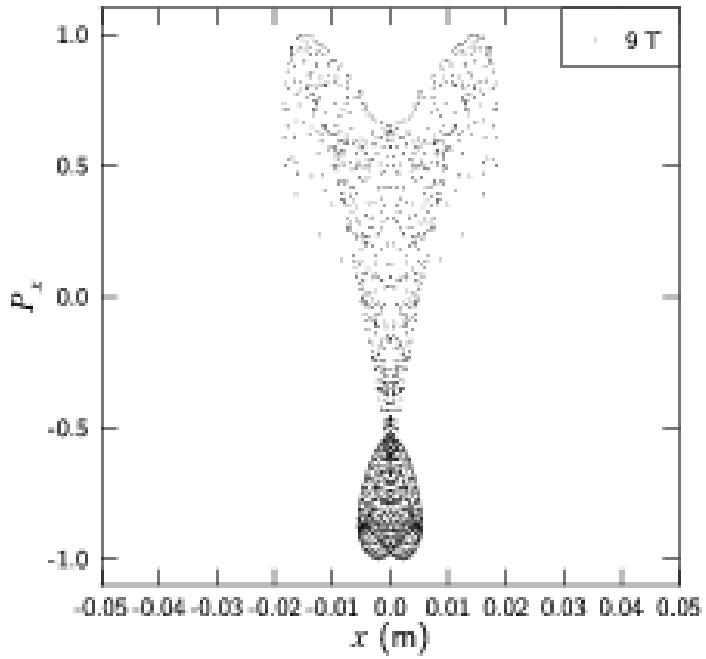
Polarization initially along x

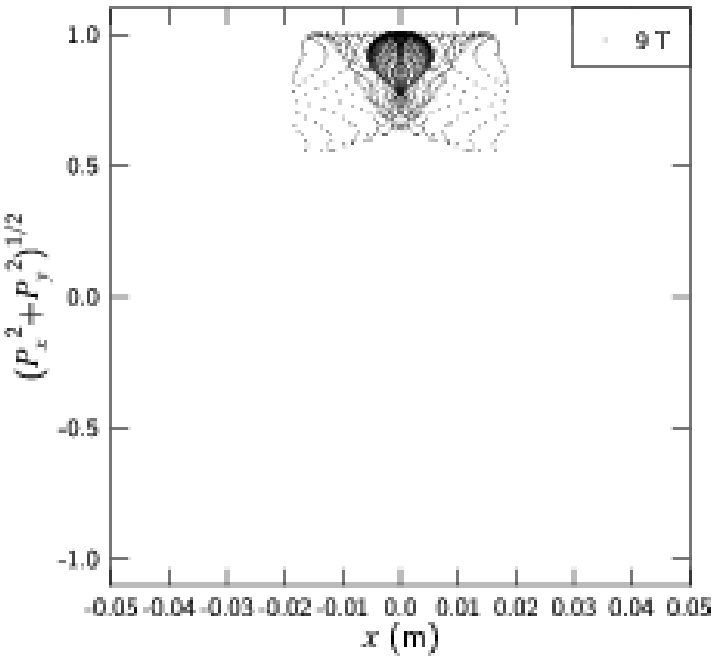
Beam radius 3 cm

$B = 9$ T





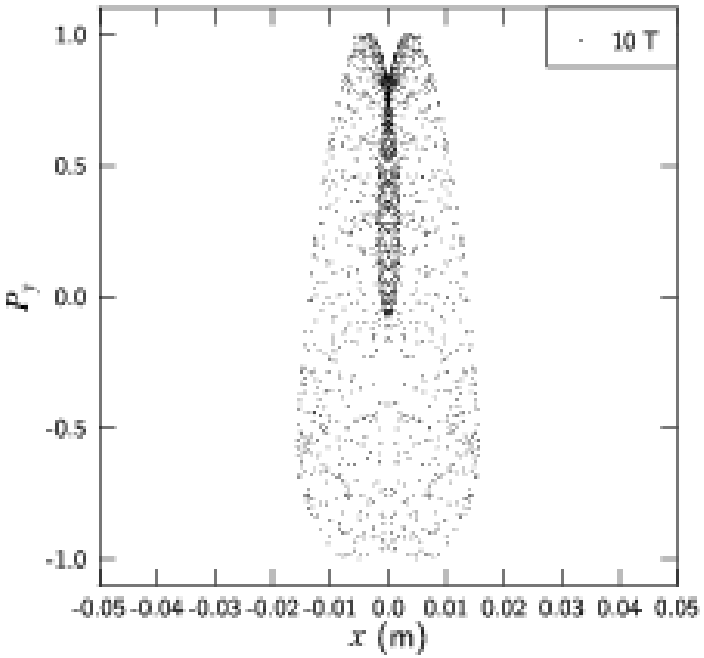


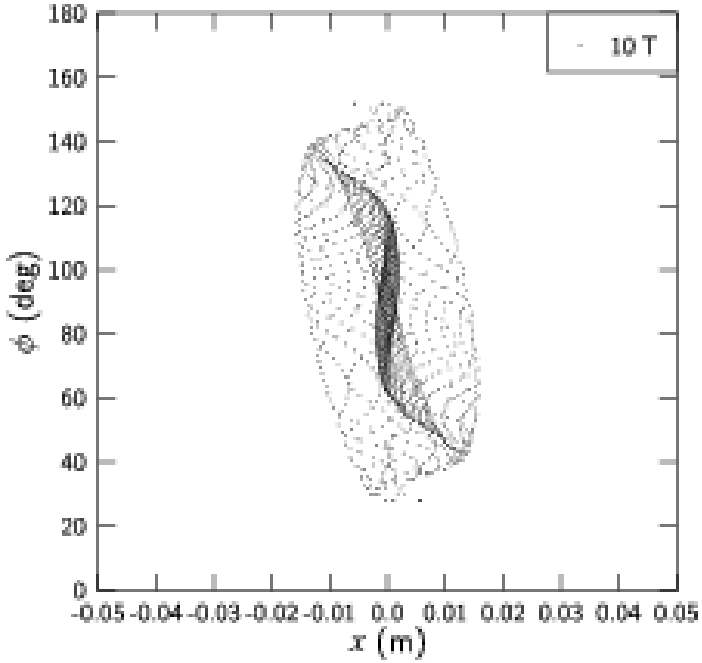


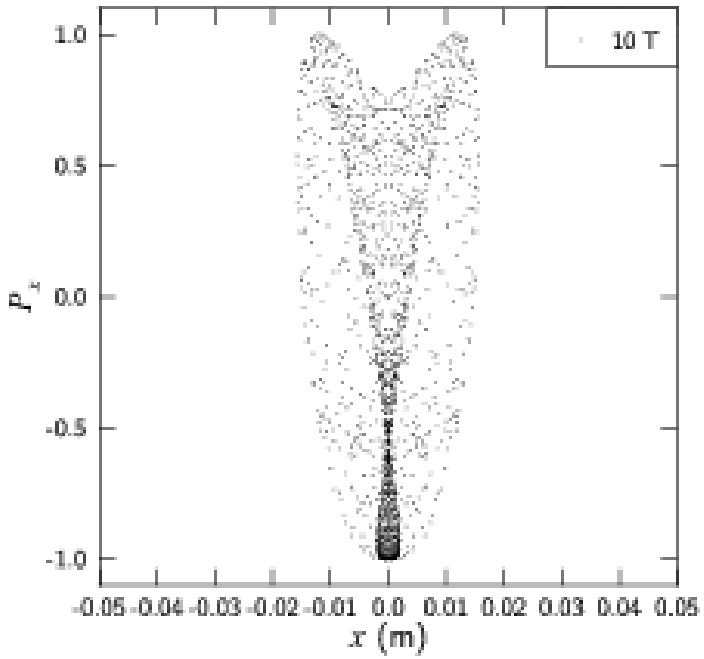
Polarization initially along x

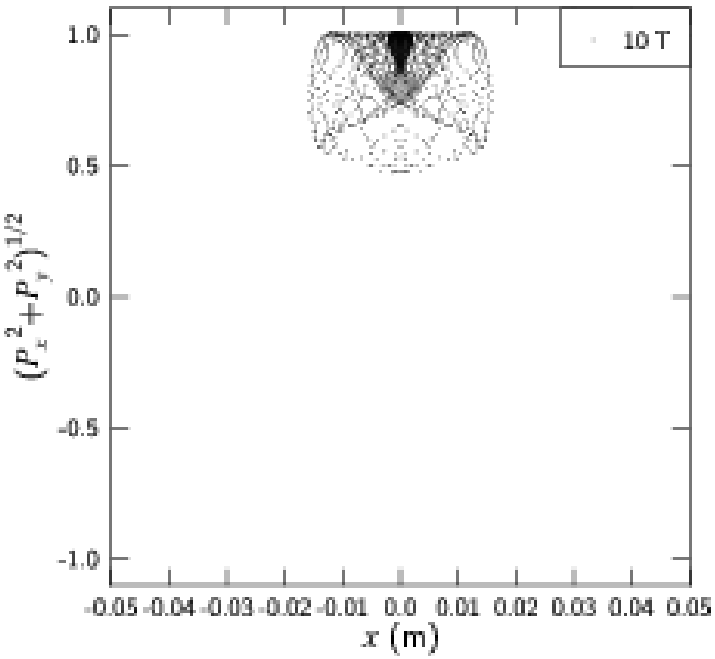
Beam radius 3 cm

$B = 10$ T



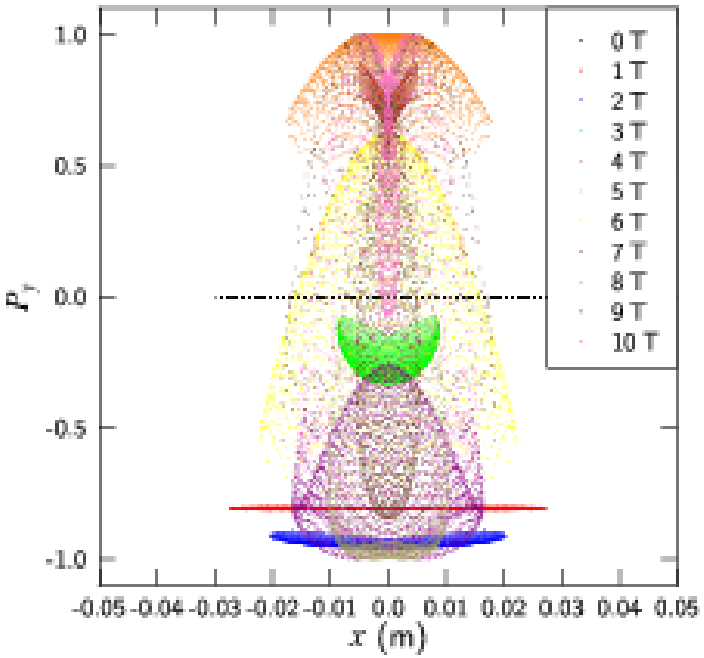


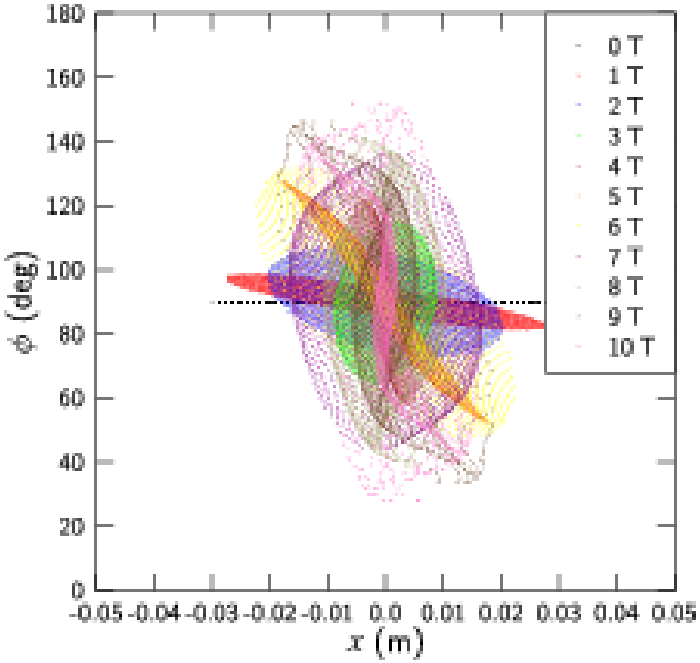


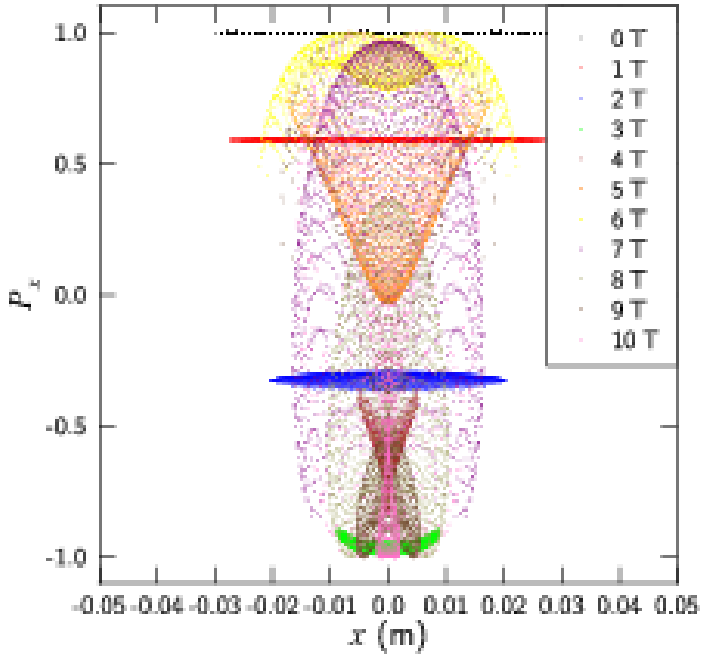


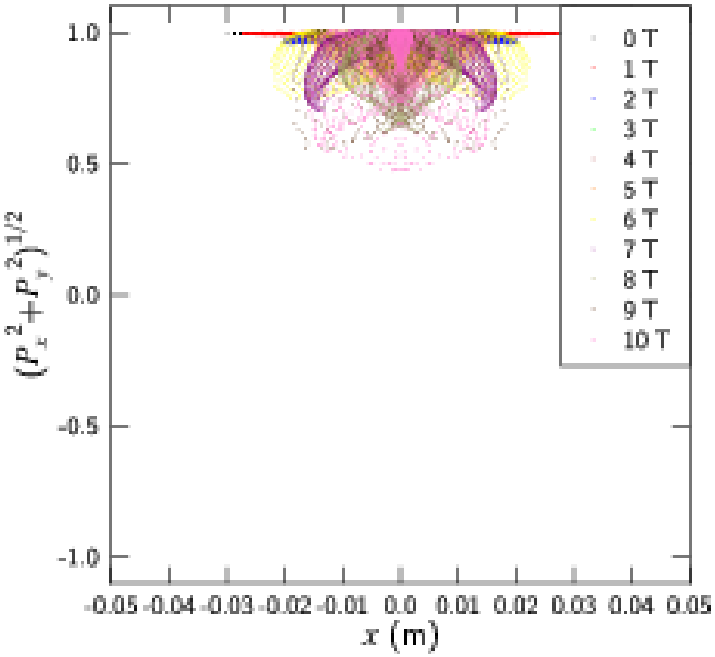
Polarization initially along x

Beam radius 3 cm





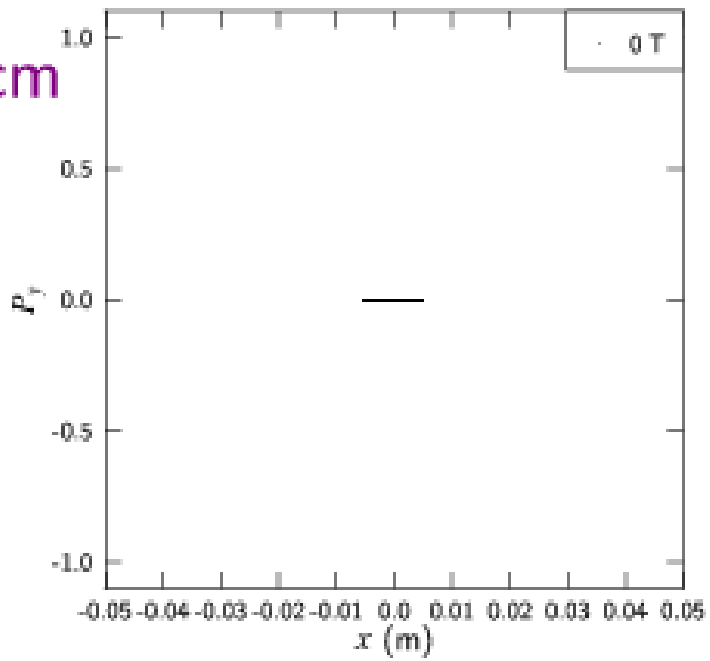


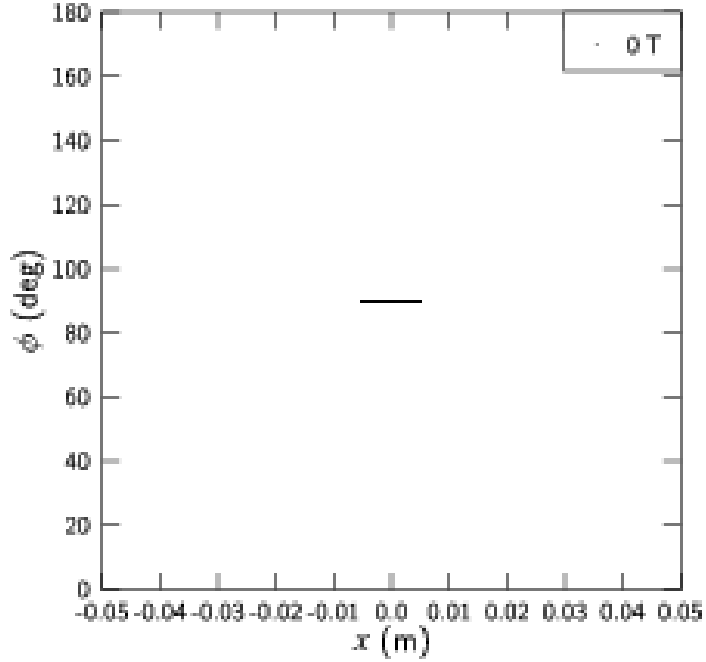


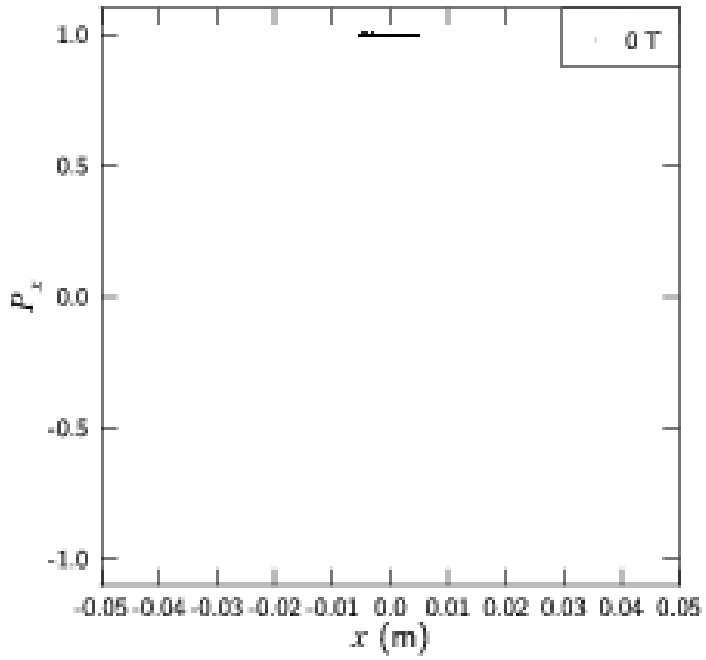
Polarization initially along x

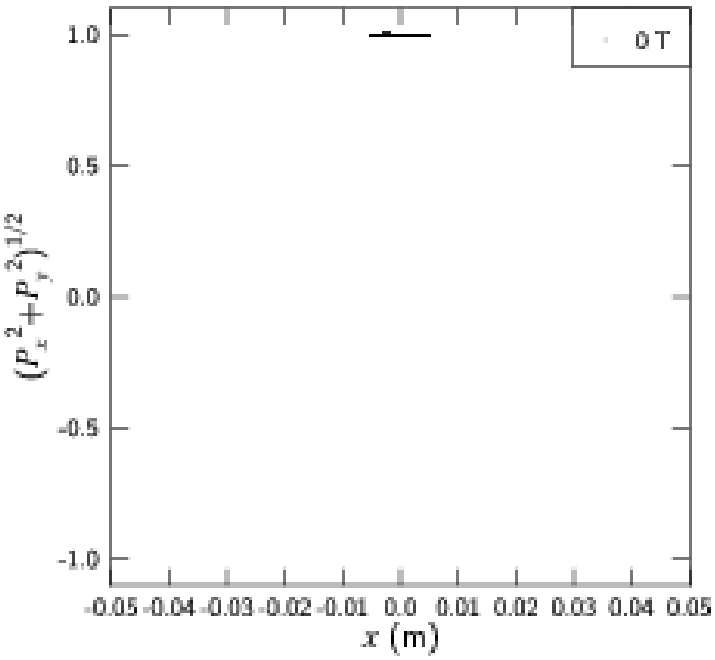
Beam radius 0.5 cm

$B = 0$ T





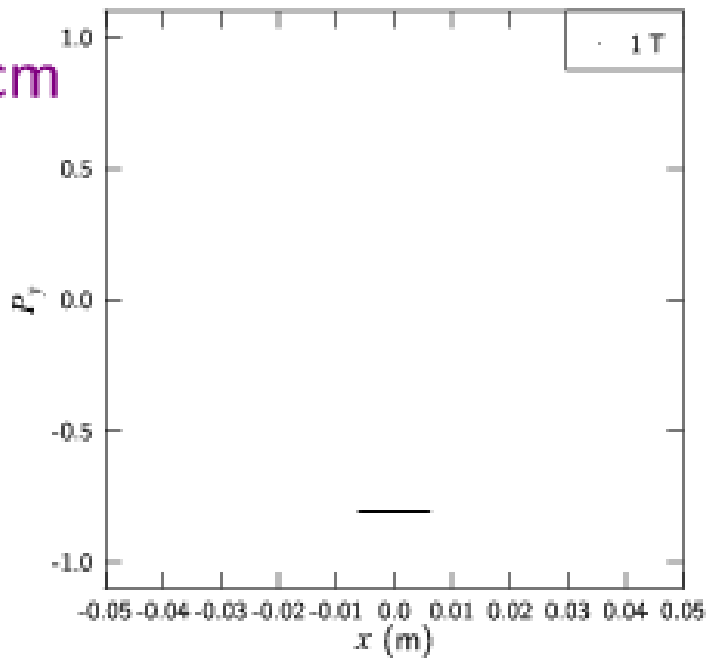


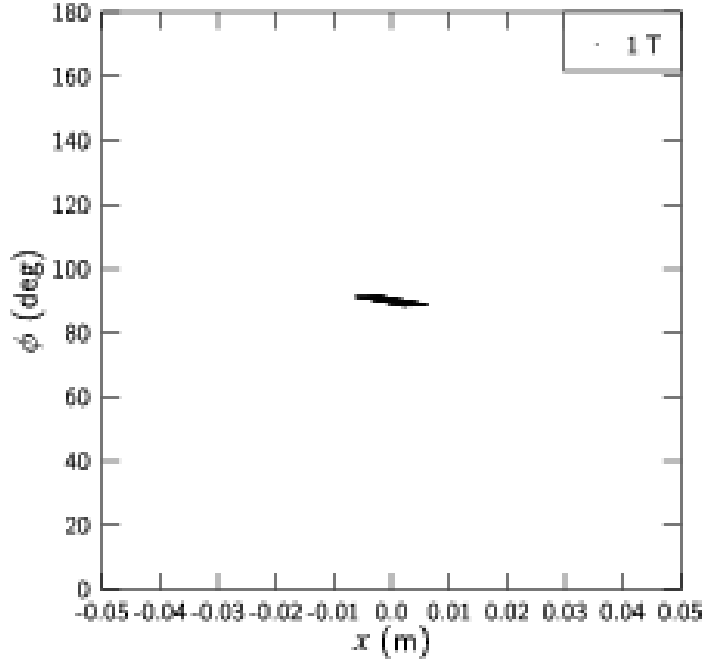


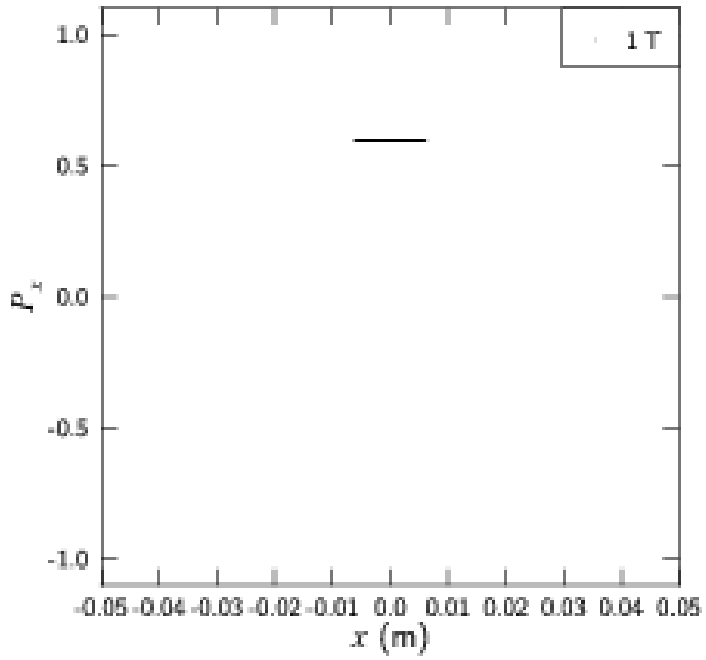
Polarization initially along x

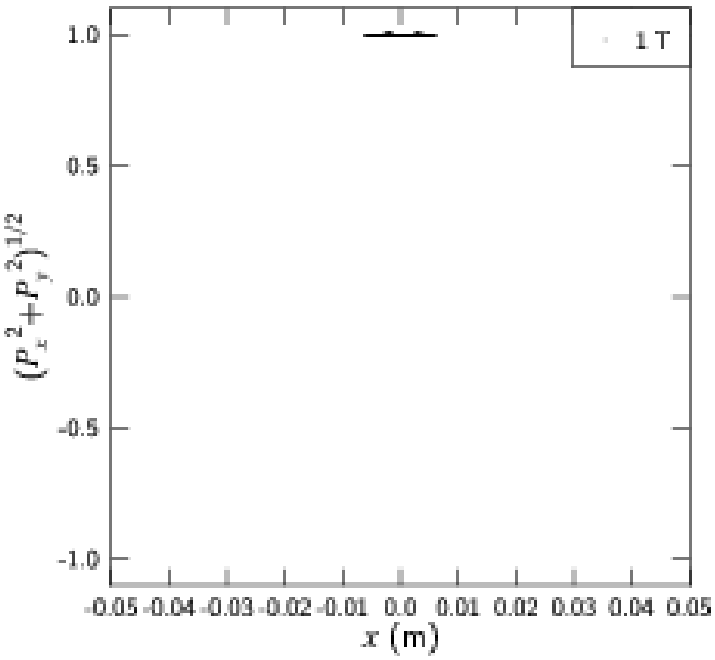
Beam radius 0.5 cm

$B = 1$ T





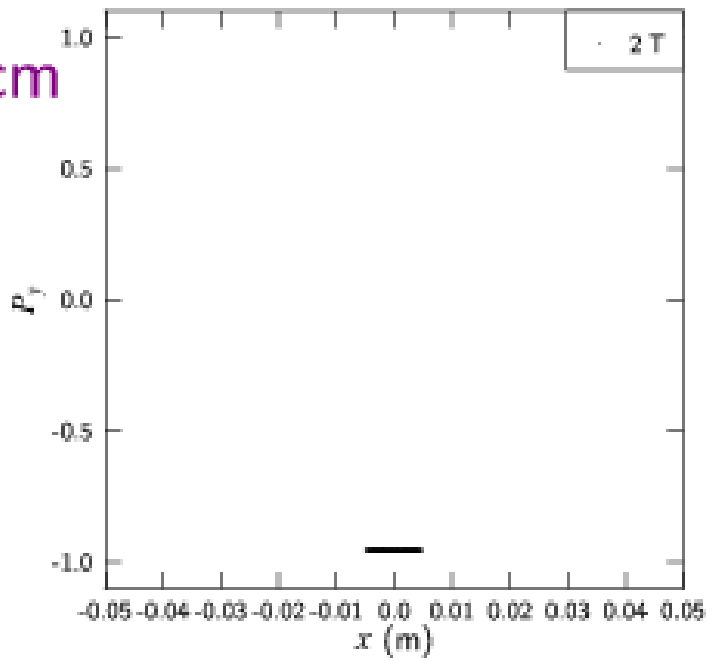


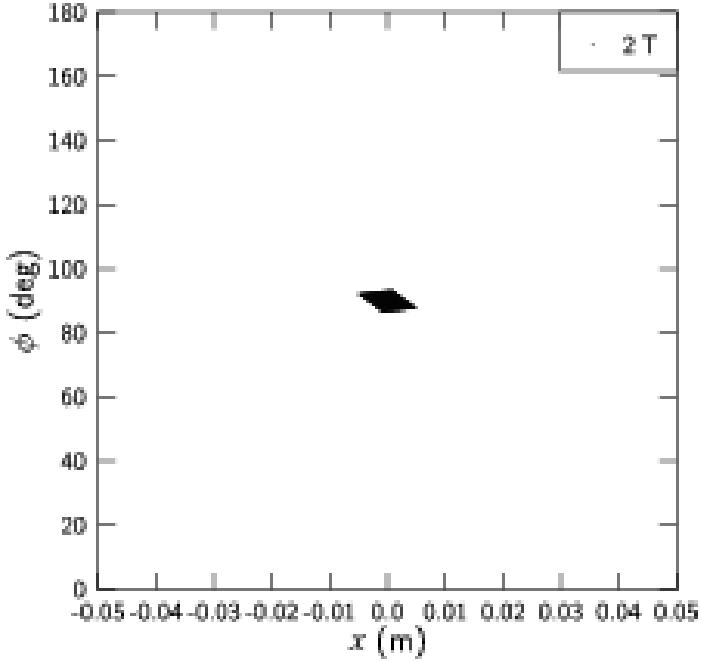


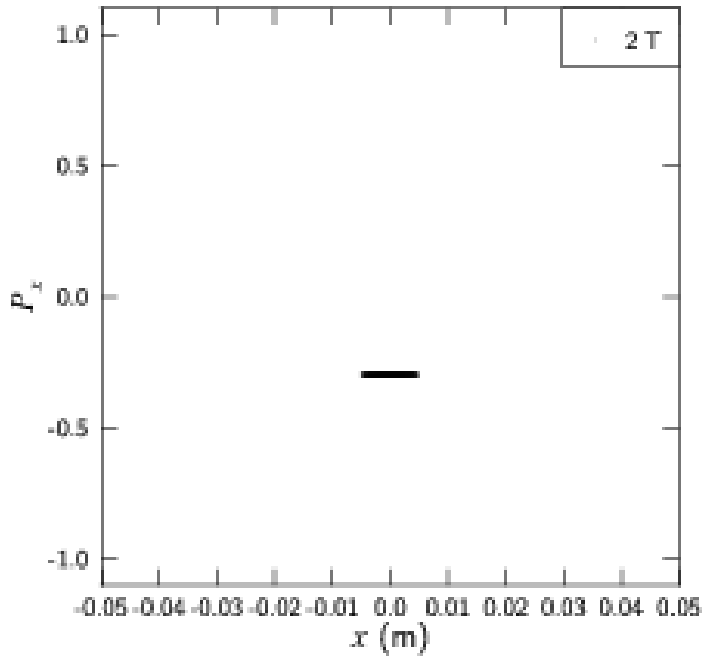
Polarization initially along x

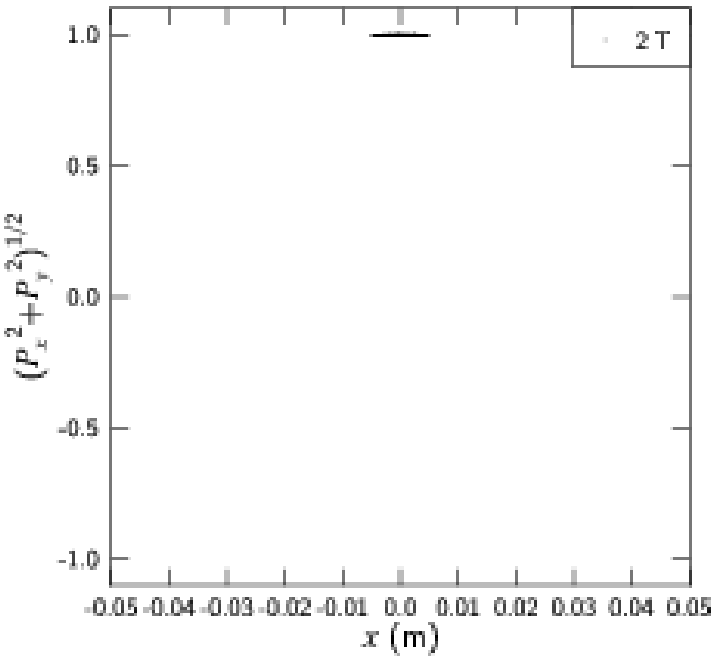
Beam radius 0.5 cm

$B = 2$ T





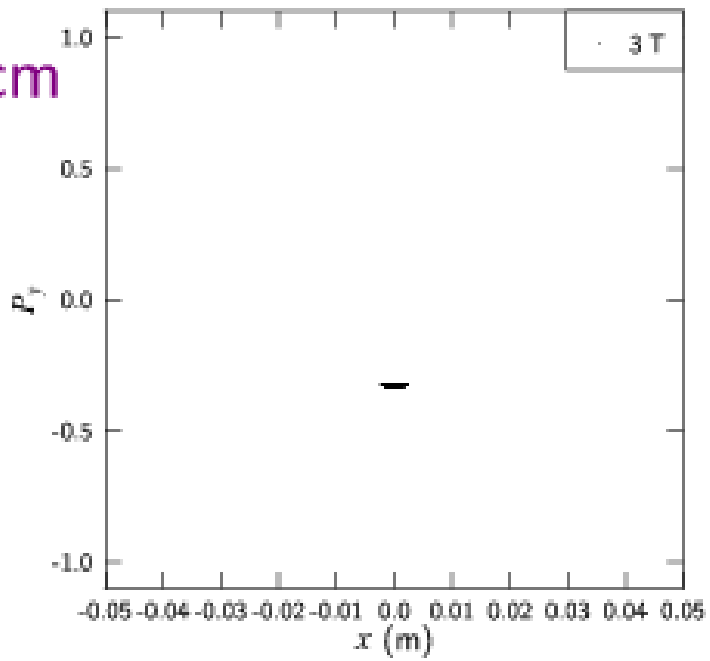


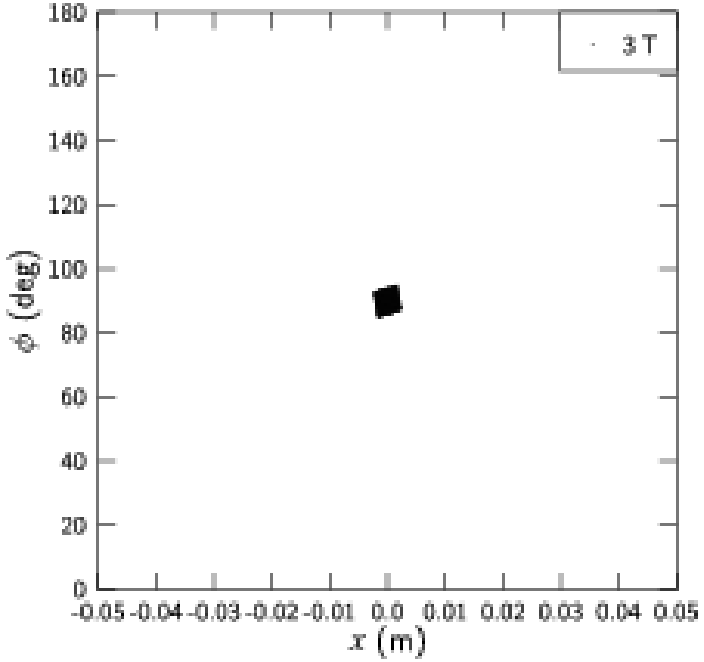


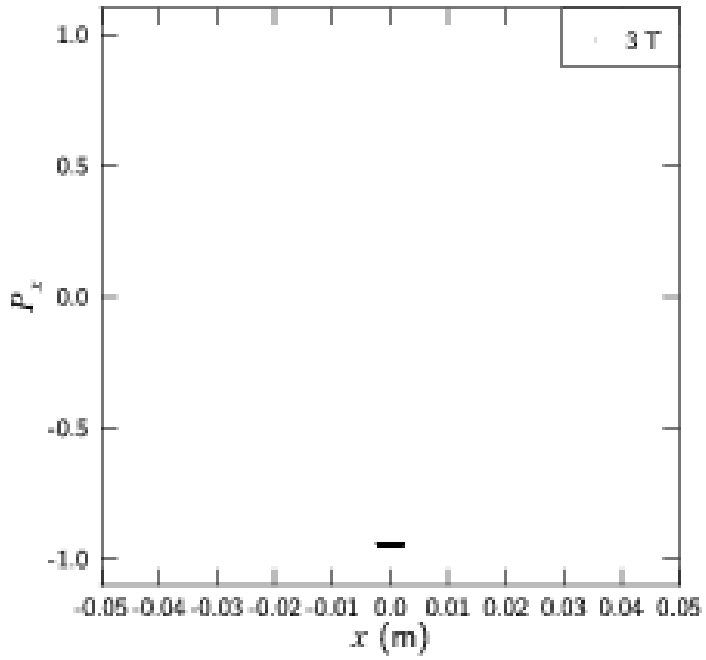
Polarization initially along x

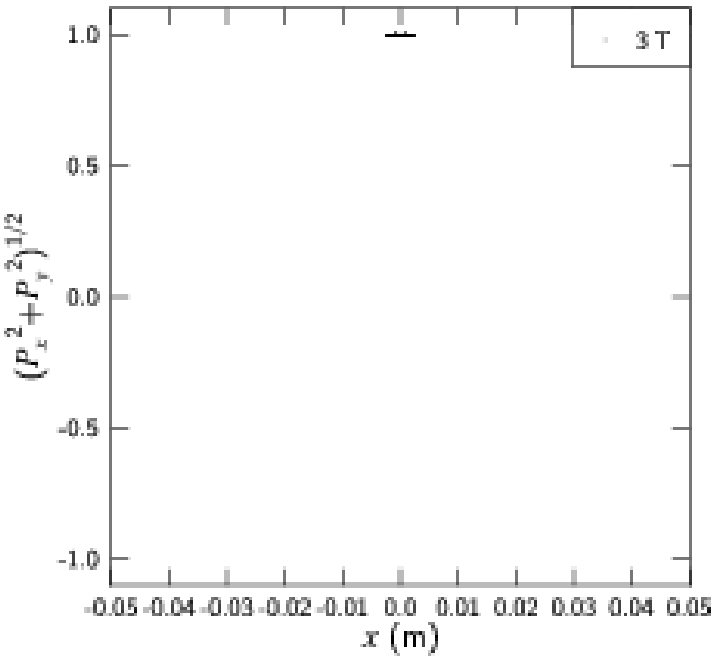
Beam radius 0.5 cm

$B = 3$ T





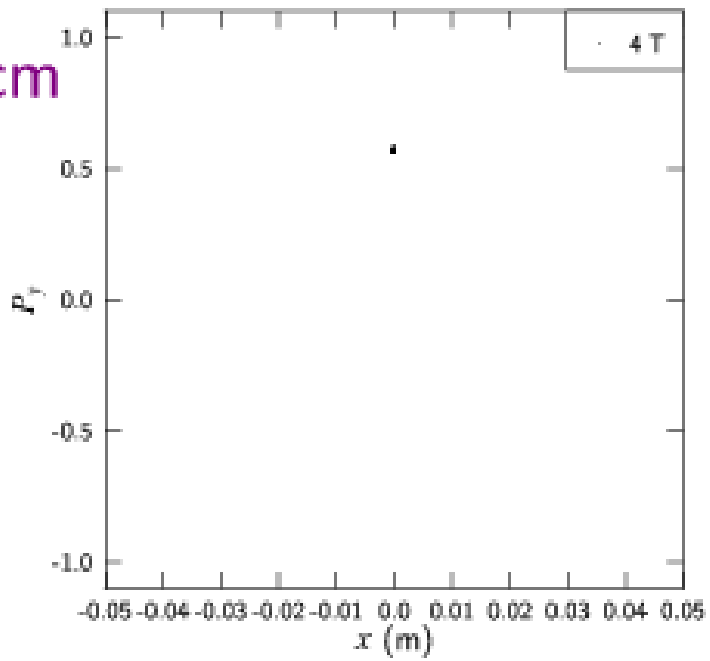


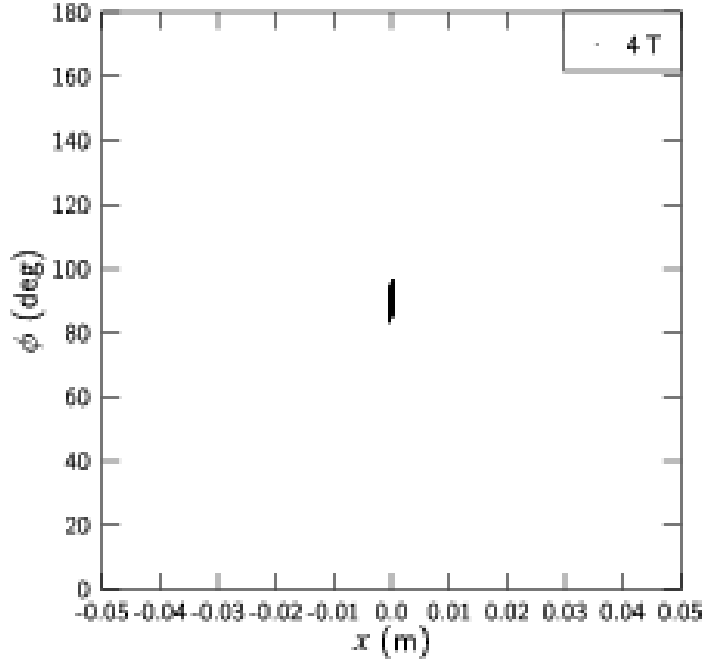


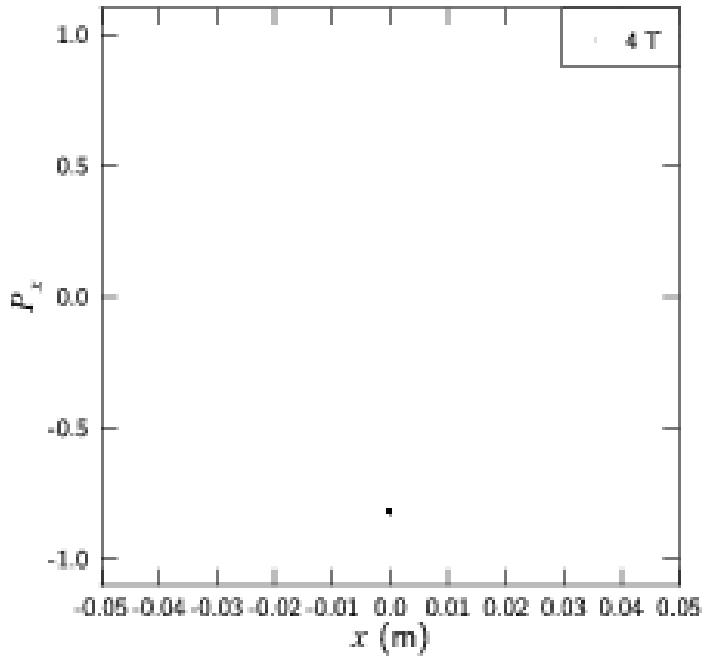
Polarization initially along x

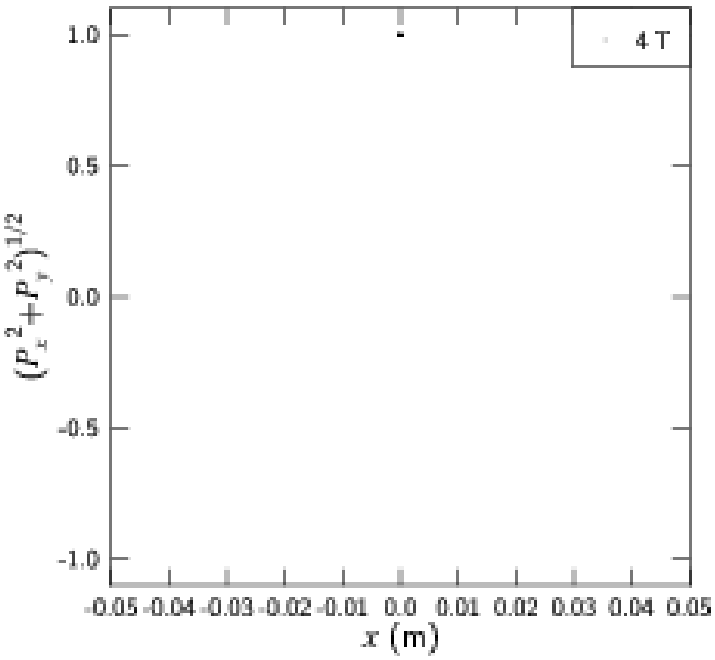
Beam radius 0.5 cm

$B = 4$ T





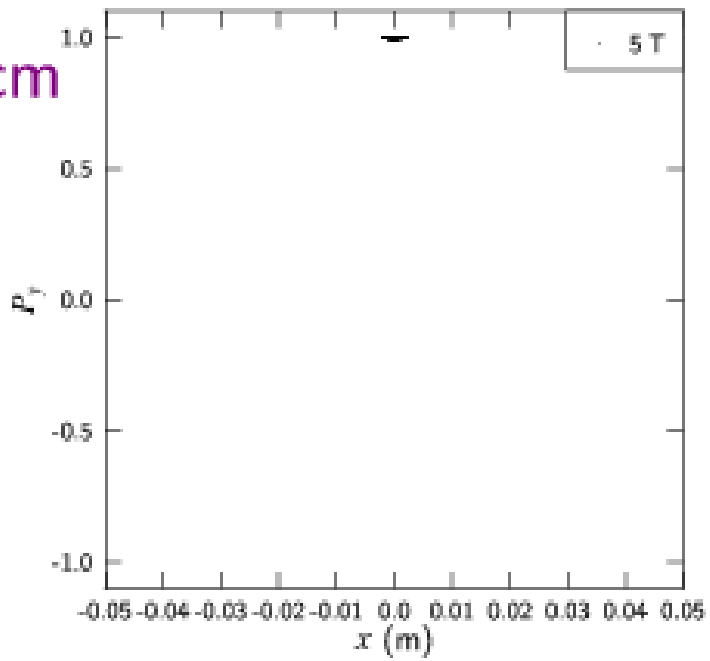


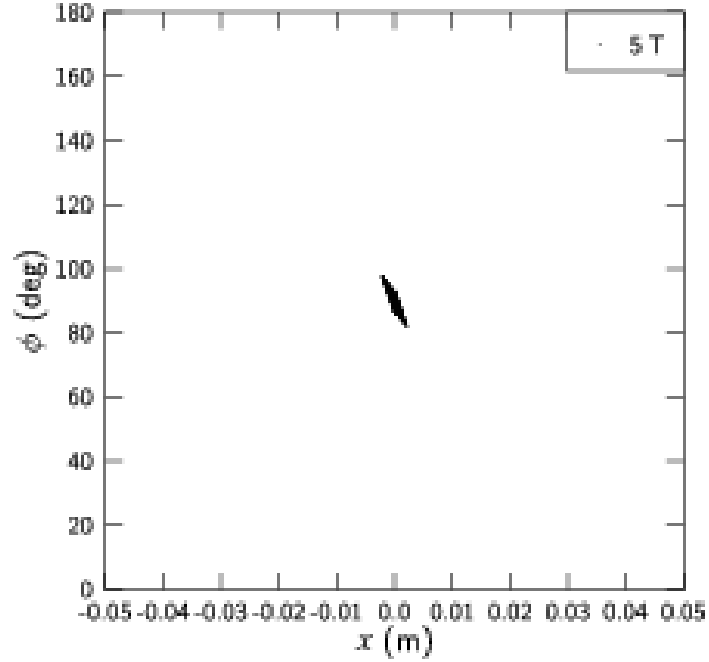


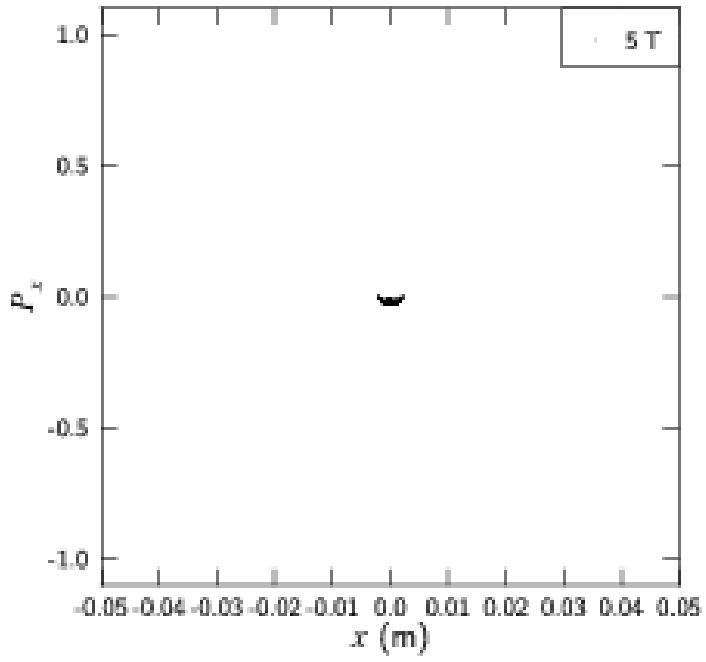
Polarization initially along x

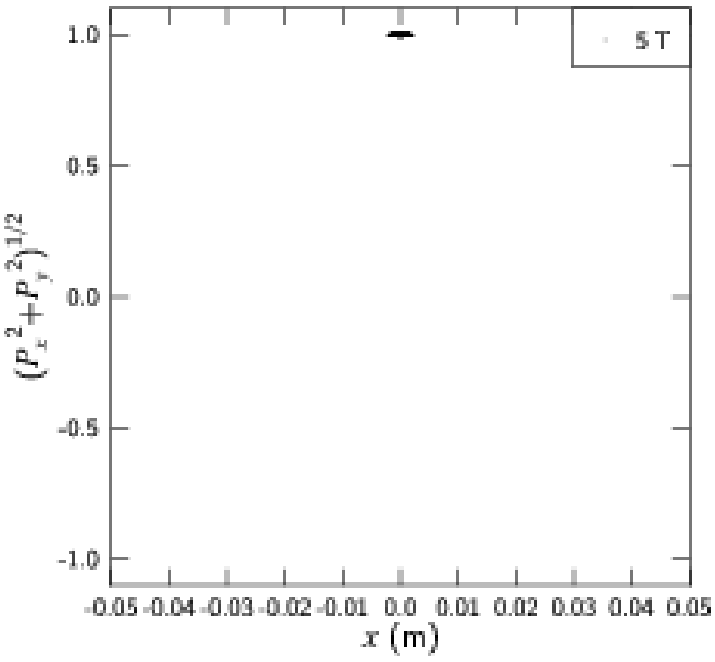
Beam radius 0.5 cm

$B = 5$ T





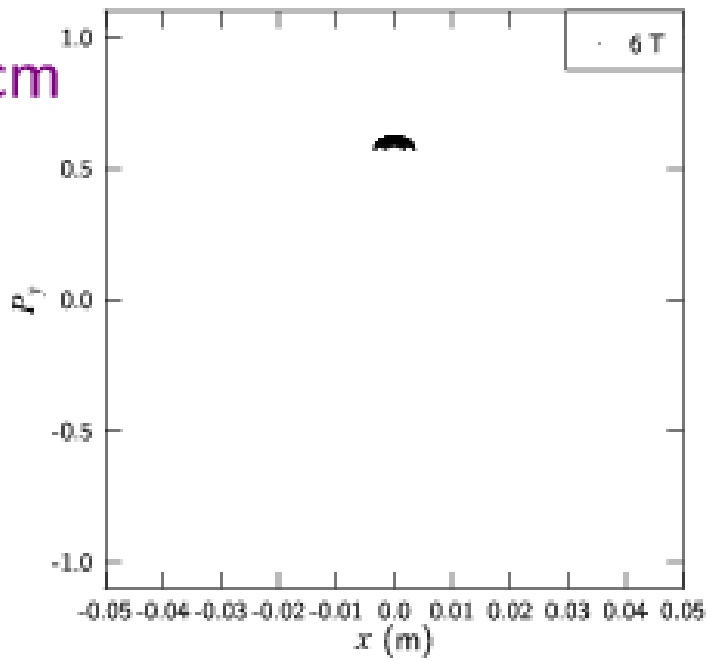


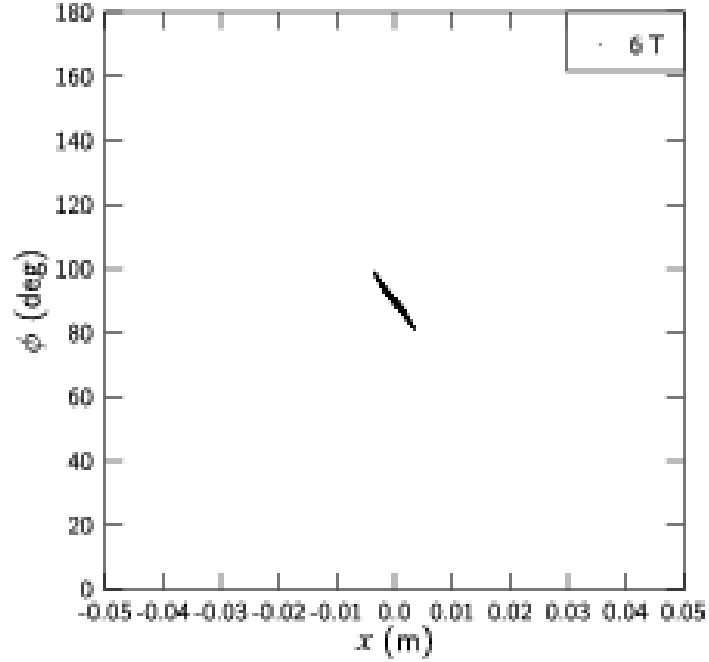


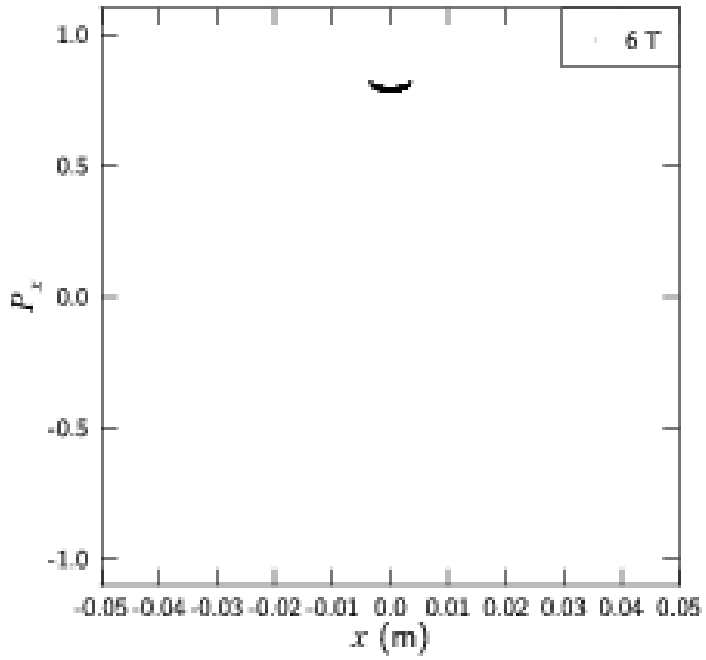
Polarization initially along x

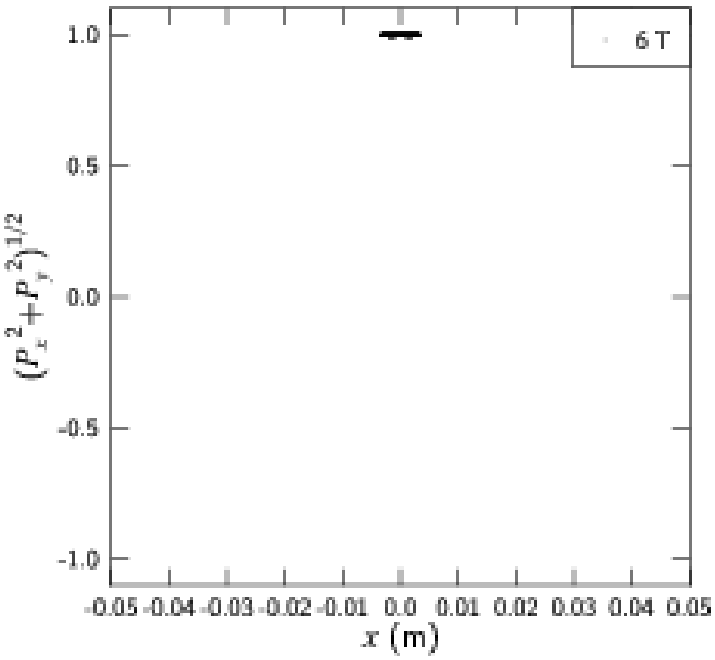
Beam radius 0.5 cm

$B = 6$ T





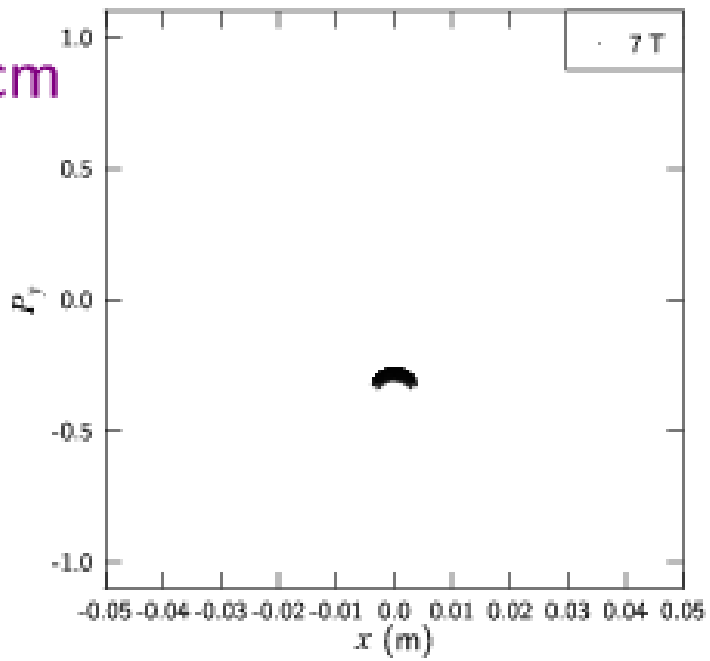


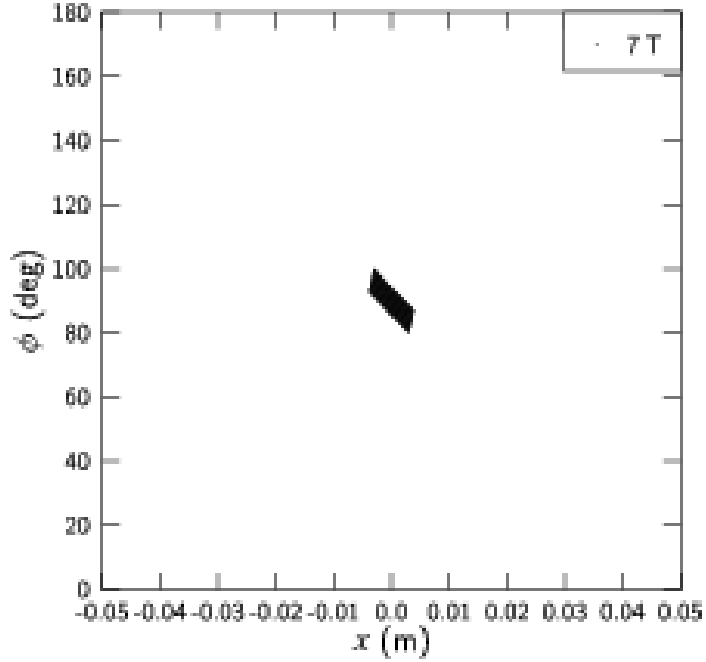


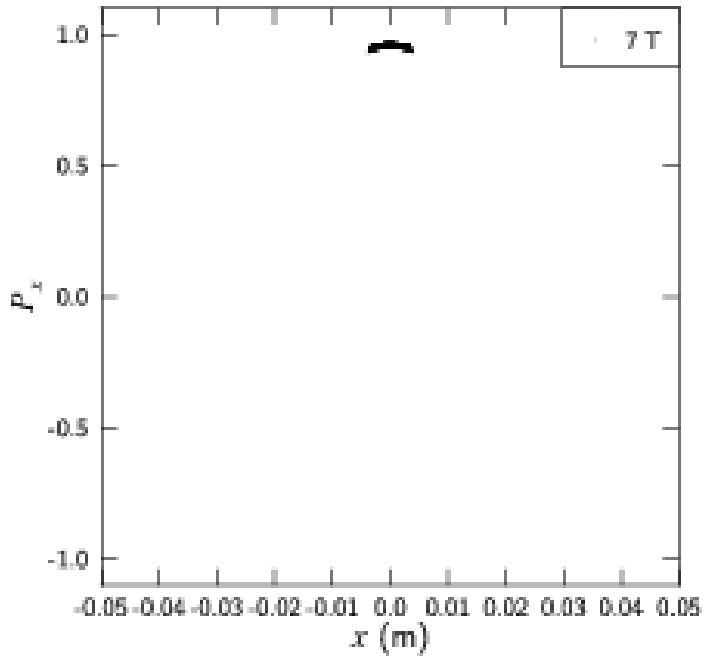
Polarization initially along x

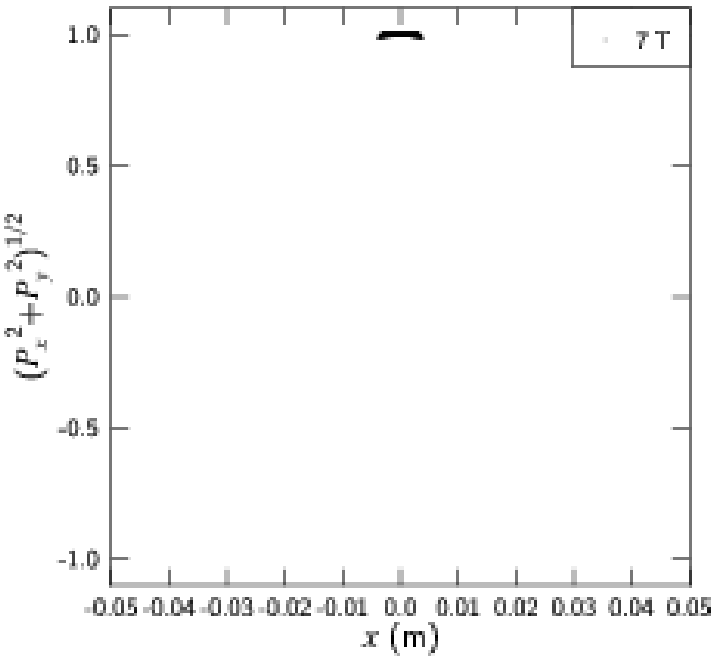
Beam radius 0.5 cm

$B = 7$ T





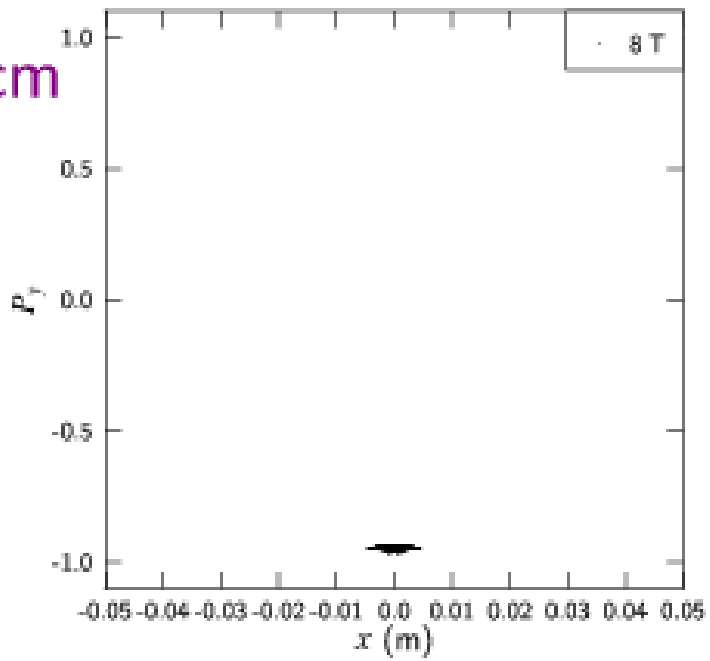


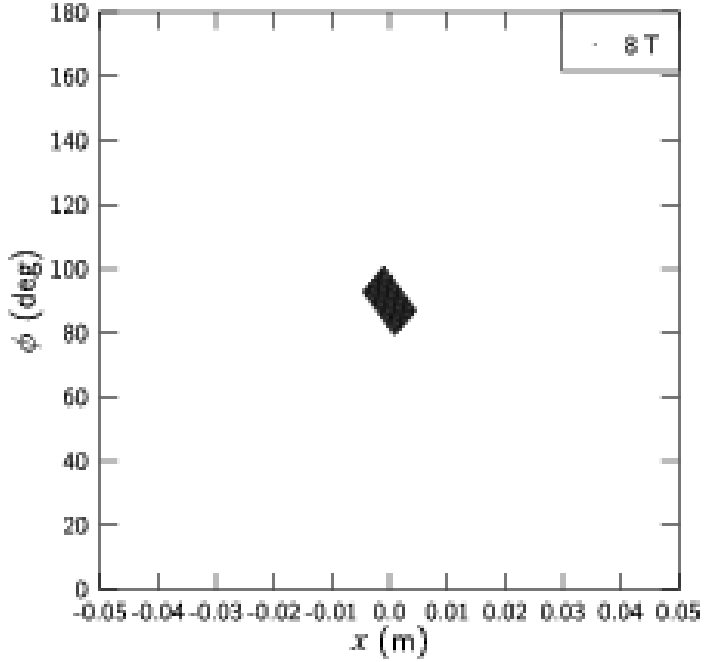


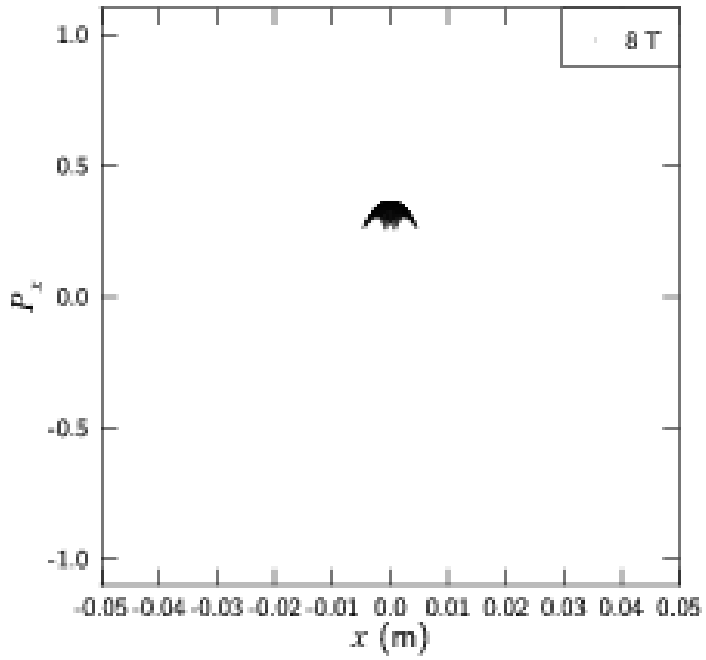
Polarization initially along x

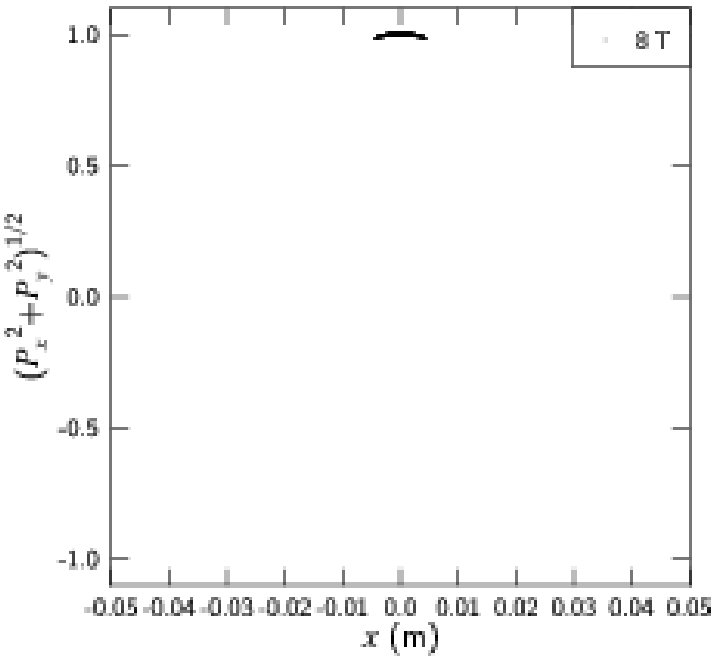
Beam radius 0.5 cm

$B = 8$ T





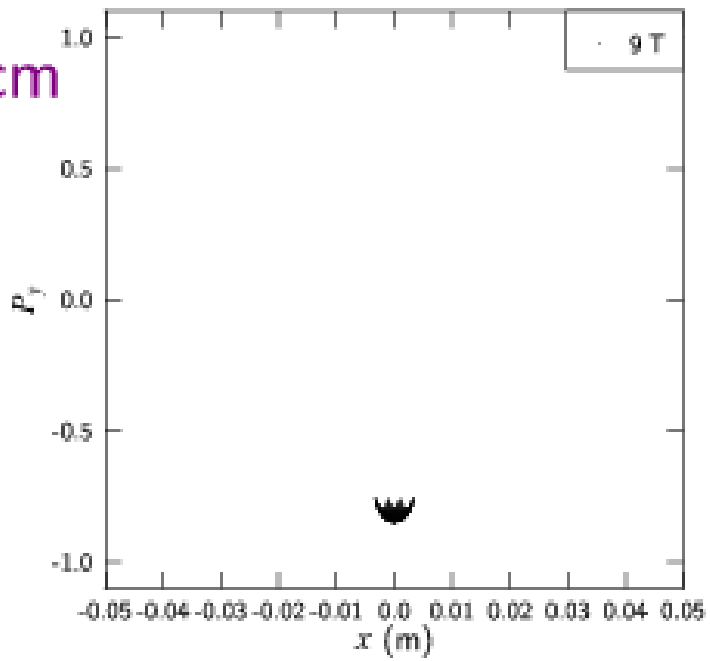


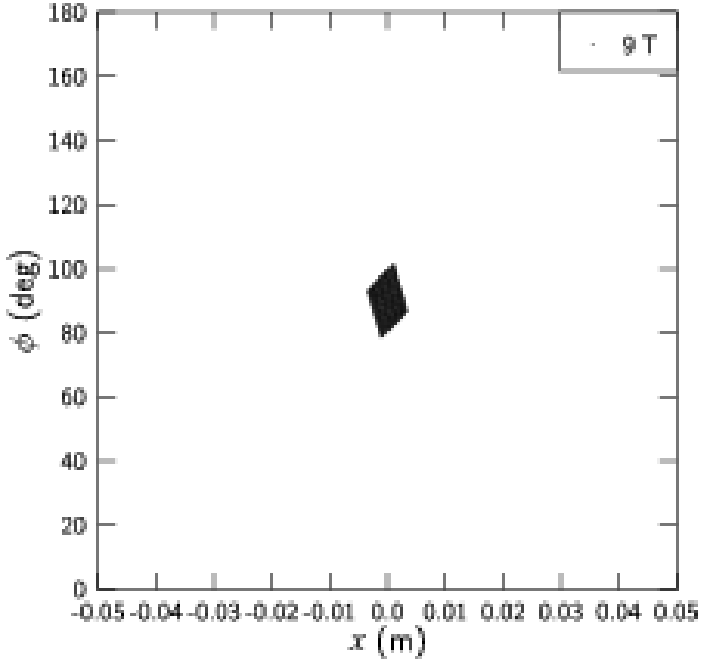


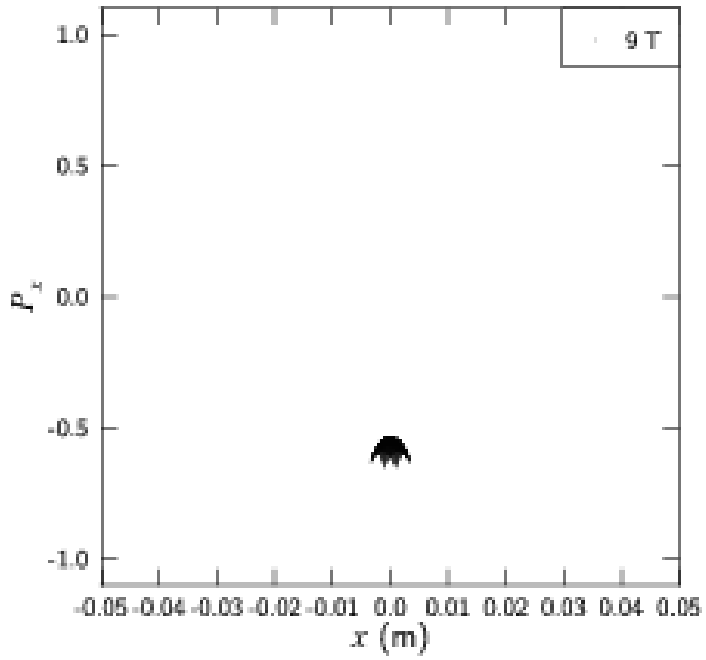
Polarization initially along x

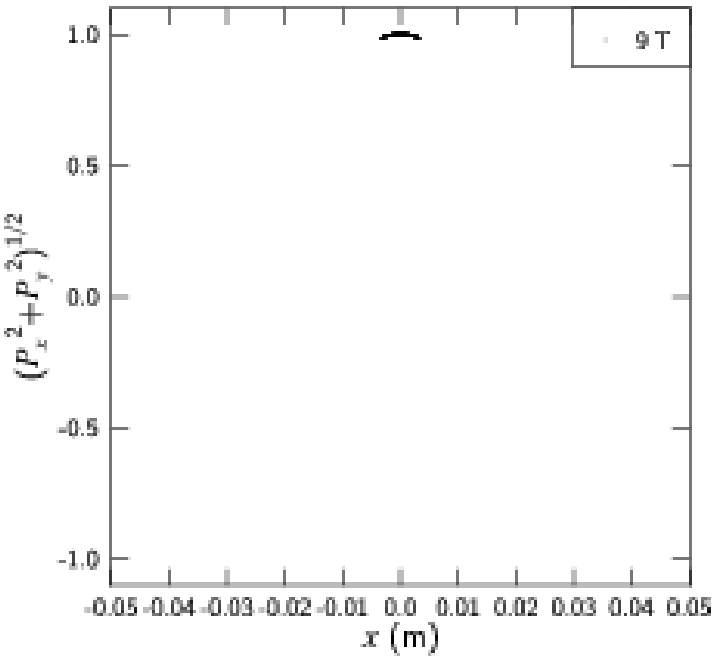
Beam radius 0.5 cm

$B = 9$ T





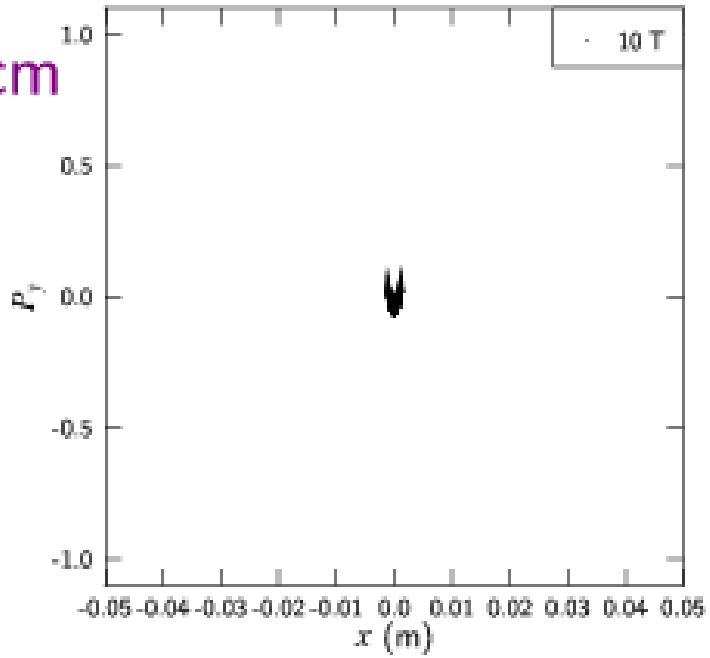


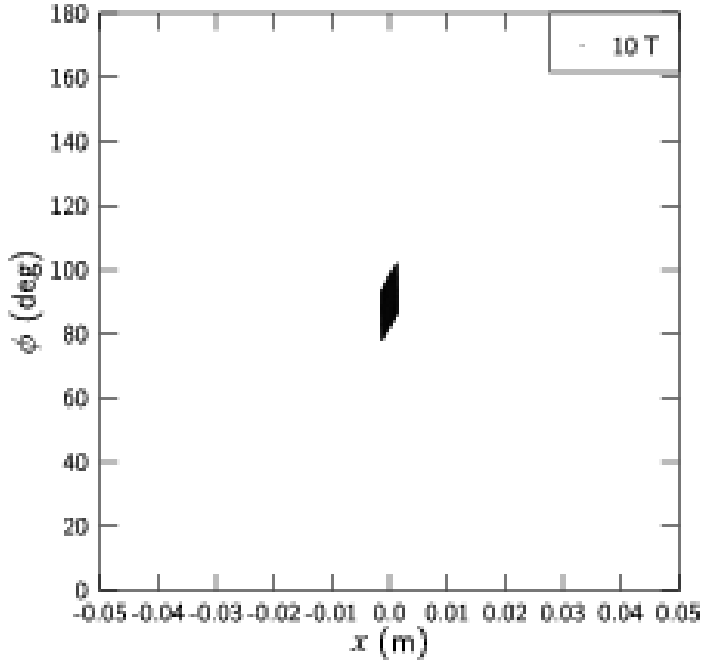


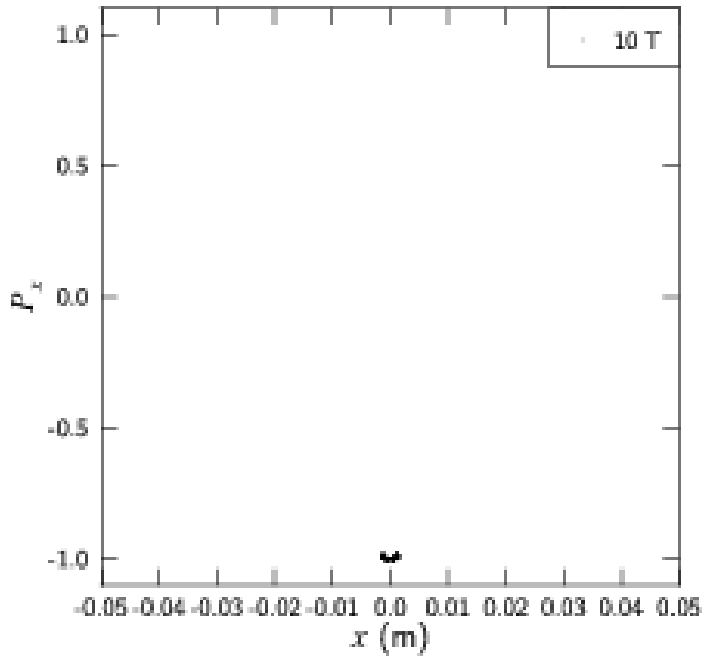
Polarization initially along x

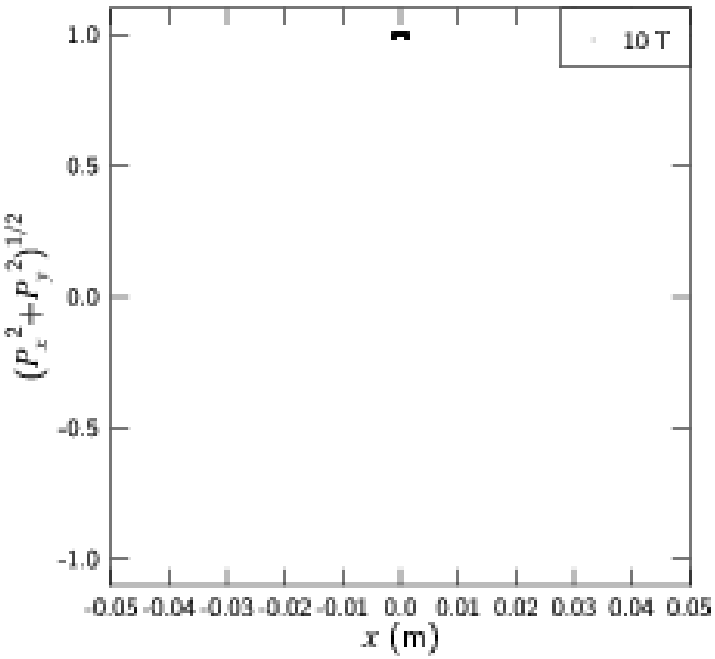
Beam radius 0.5 cm

$B = 10$ T



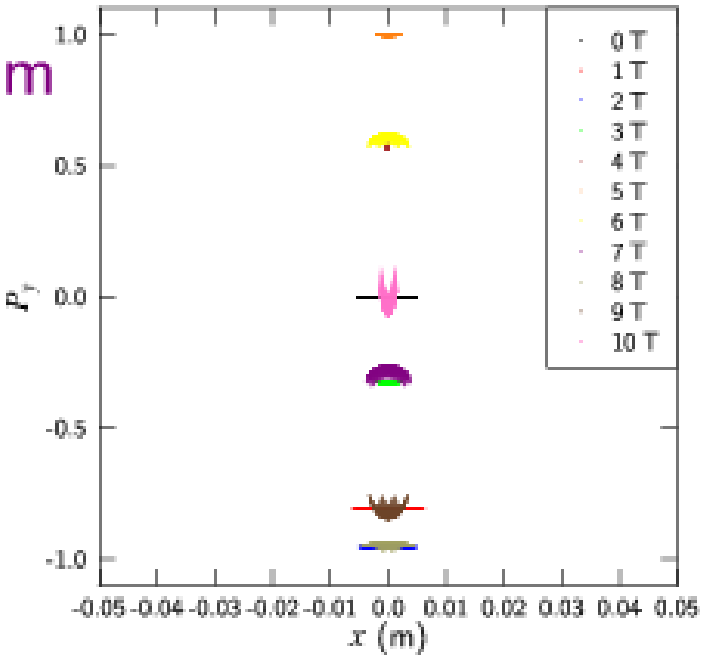


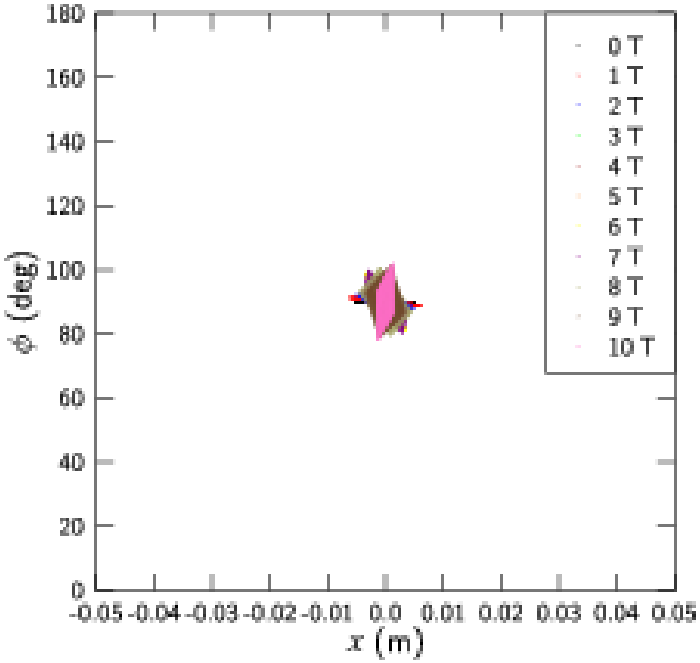


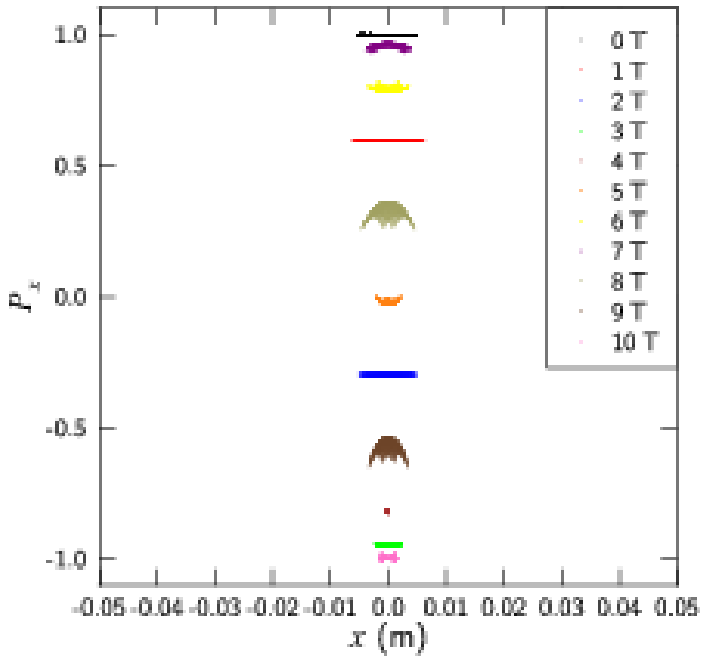


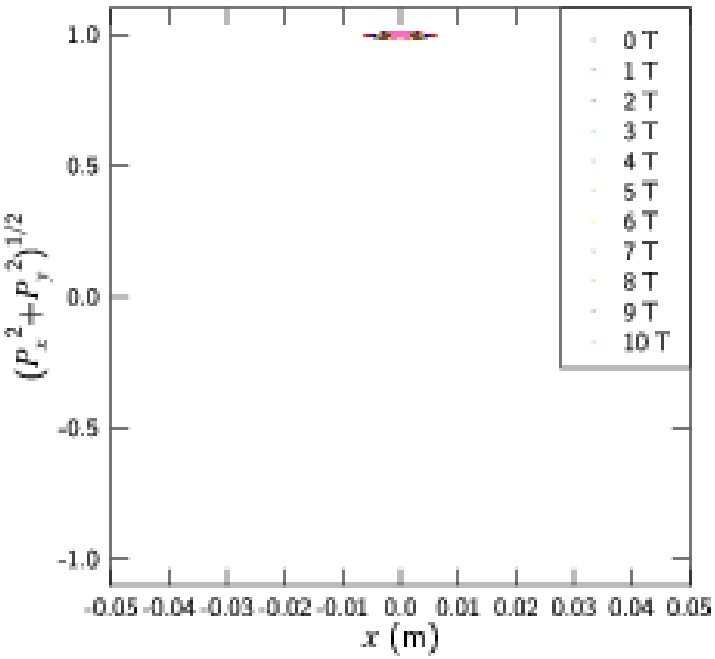
Polarization initially along x

Beam radius 0.5 cm

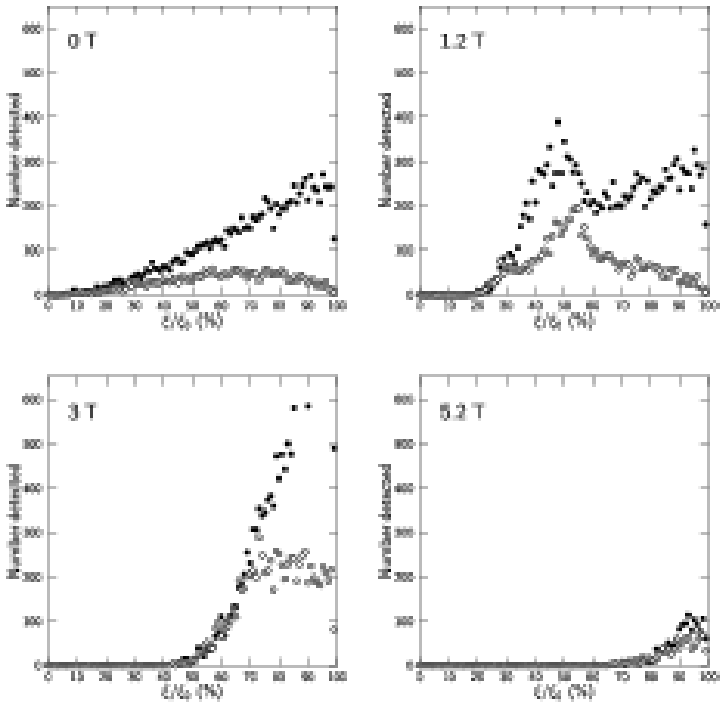








Decay positrons



Considerations

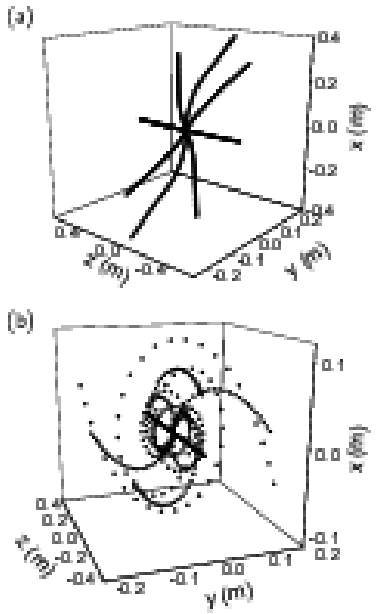
Lighter and faster particles

Relativistic corrections more crucial

Other important physical processes to include?

Muon decay has been implemented in both program packages

Tests show initial agreement



Must now consider detector arrangements

Towards full instrument simulation

Next step: full simulation of ALC?

T Lancaster, D.Phil Thesis, Oxford (2004)

Conclusions

Simulations have been carried out using Tofu and Geant4

Incoming muon trajectory and polarization has been calculated for fields from ALC and new magnet design from Cryogenic

Muon decay has been implemented in both packages

Further tests of positron trajectories in progress

Next step: full instrument simulation